

## **The Influence of Hiatuses on Sediment Accumulation Rates**

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### **Abstract**

Hiatuses pervade the stratigraphic record at all scales from grain boundaries to inter-regional unconformities. Every attempt to measure a rate of accumulation must average together sediment increments and surfaces of hiatus. As the time span of measurement lengthens, longer hiatuses tend to be incorporated into the estimated rate. Consequently, short term rates are systematically faster than longer term rates. As would be expected for a fractal time series, the empirical relationship between accumulation rate and the time span of measurement is a negative power law. Just as a measured length for a coastline is meaningless without a statement of the map scale, a measured rate of sediment accumulation requires a statement of the time scale of measurement. Fractal scaling laws offer a means to estimate and interpret the changes in average accumulation rate from one time scale to another.

The steepness of the negative power law that relates rate to time span increases with the incompleteness of the stratigraphic record. For steady accumulation and complete sections the slope would be zero. For the most incomplete and unsteady accumulation, the slope approaches minus one. A purely random accumulation process produces a power law with a slope of minus one half; less steep negative slopes indicate some degree of persistence or positive correlation between successive sediment increments; steeper negative slopes prove a negative correlation. Regular periodic fluctuations in accumulation are one form of negative correlation; they produce steep negative slopes at time scales close to the period of the fluctuations. In shallow marine carbonate environments the negative power laws steepen markedly at time spans of tens of thousands to hundreds of thousands of years; the time scale suggests that the steepening records the preponderance of hiatuses that have been generated by Milankovitch-scale changes of sea-level.

A plot of the age and level of sediment increments preserved in a stratigraphic section has a staircase-like shape, in which the treads represent surfaces of hiatus. Sequence boundaries of known age fix the coordinates of points on these treads. Rocks dated by radiometric techniques fix the coordinates of points on the risers. Horizons across which the fauna, isotopic composition or magnetic polarity are found to change may correspond to points on either the treads or the risers. The staircase plot forms the lower bound of the true accumulation history which would be determined by continuous monitoring of the elevation of the sediment surface during accumulation. Thus, the relationship of accumulation rate to time span also depends to some extent upon the method used to date the sediment.

### **Rates and Unsteady Processes**

Accumulation rate has a deceptively simple formulation -- measure the thickness of the sedimentary deposit and divide by the time that elapsed between the start and finish of deposition. Thus, changes in thickness of a deposit or changes in the elevation of the sedimentary surface are, in effect, standardized for differences in the time span of observation. Nevertheless, a measured accumulation rate has no general power to convert deposit thickness to duration of deposition, or stratigraphic elevation to age. If sediment accumulation were steady, then a rate determined from any portion of the accumulation history (regardless of age and duration) would indeed be the correct conversion factor for every other portion. We could interpolate age linearly between dated horizons and one measured rate would support extrapolation beyond dated intervals. But sedimentation is, of course, inherently unsteady and discontinuous. Surfaces of hiatuses record the discontinuities.

This paper will show that the presence of hiatuses causes the expected accumulation rate to become a strong negative function of the time span of measurement. Empirical data demonstrate that rates measured at one time span cannot be extended with impunity to other time spans anywhere in the range from minutes and hours to hundreds of millions of years. The underlying principle becomes almost intuitively obvious when we consider the deposition of sediment as a succession of instantaneous arrivals of discrete particles with finite diameters. Each particle diameter is added to the net thickness at an instant in time. Consequently, instantaneous accumulation rates can be infinitely high. Of course, nobody would consider applying such instantaneous accumulation rates to longer time intervals of geologic interest. But rates measured on the relatively short time spans of human observation are routinely applied to much longer stratigraphic time scales. The conceptual error is the same as extrapolating from the instantaneous rates. Rates from different time scales are not directly comparable and hiatuses are the cause, whether the hiatuses are recorded at the scale of grain-to-grain boundaries or inter-regional unconformities.

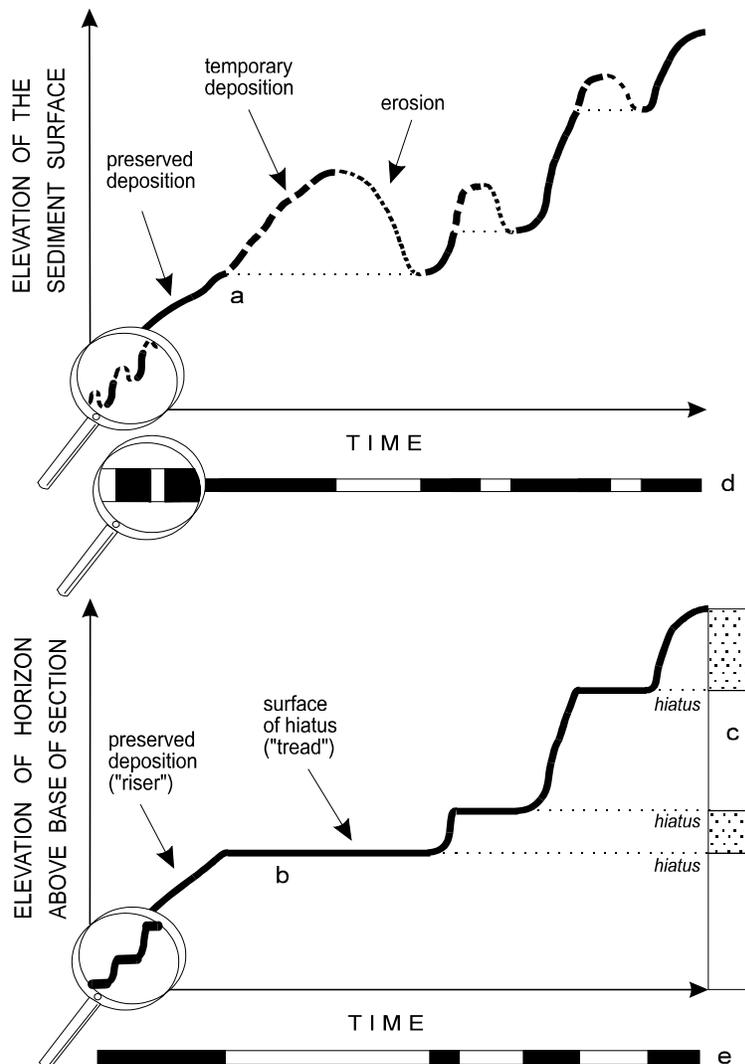
Fortunately, fractal scaling laws offer a means to estimate and interpret the changes in average accumulation rate from one time scale to another. After establishing how accumulation rates scale with time span for a variety of depositional environments, the paper reviews some standard results from fractal mathematics that help to relate the scaling laws to properties of the underlying geologic processes. Then, two examples serve to demonstrate how the failure to appreciate the time-scale dependence of rates does lead to false geologic interpretations. Next, we examine how the scaling laws can vary with the method of determination of age. With these insights, the paper concludes by analyzing a single stratigraphic section that has been dated by both radiometric and paleomagnetic methods. Before any of this, however, it is useful to review the nature of hiatuses and the options for displaying stratigraphic data.

### **The Pervasive and Composite Nature of Hiatuses**

Hiatuses pervade the stratigraphic record at all spatial scales from grain boundaries to bedding planes to inter-regional unconformities [1,2]. Surfaces of hiatus record time intervals without net deposition. The duration of these intervals ranges from the fractions of a second between successive grain arrivals at the sediment surface to the millions of years that may elapse between marine flooding events on continental interiors. Tiny hiatuses must be pervasive because sediment consists of discrete particles. The areal extent of intergranular hiatuses must be extremely limited. Bedding planes and laminae record innumerable discontinuities in accumulation at larger spatial scales and longer times scales. And practitioners of sequence stratigraphy have advanced the case that inter-regional hiatuses are sufficiently commonplace and areally extensive to provide a basis for global time correlation.

During the time interval in which a surface of hiatus is generated, deposition may simply have ceased or erosion may have removed some of the previously deposited sediment. Consequently, three distinctly different components of the accumulation history may all be condensed into a single

surface of hiatus: episodes of non-deposition; episodes of erosion; and time intervals occupied by the accumulation of sediment that is subsequently eroded [3]. The closing sections of this paper explore the fact that some methods used to determine the age of sedimentary deposits can produce dates that fall within these intervals of non-deposition, erosion, and temporary deposition that make up hiatuses.



**Figure 1.** The distinction between the accumulation history (a), staircase plot (b), columnar section (c), and time lines (d, e) for a hypothetical sedimentary deposit. Time line d distinguishes intervals of deposition from intervals of erosion. Time line e distinguishes time intervals recorded by preserved sediment increments from intervals of hiatus. The magnifying glasses caution that smaller hiatuses exist below the resolution of this diagram. In the limit of extremely fine resolution, the solid bars in time lines d and e become sets of discrete points and the risers in curves a and b become vertical.

At any given level of resolution, the sedimentary stratigraphic record may be regarded as a stack of sediment increments separated by surfaces of hiatus. But closer inspection should always reveal that the sediment increments include smaller hiatuses, down to the resolution of individual grains. Systems like this in which a common motif is nested within itself at different scales are often usefully viewed from the perspective of fractal geometry. Sediment accumulation is no exception.

Every attempt to measure a rate of accumulation averages across sediment increments and the intervening surfaces of hiatus. As the time span of measurement increases, larger hiatuses may be incorporated into the estimated rate. As the time span of measurement decreases, the revelation of smaller hiatuses requires that increasing numbers of discrete sediment increments are revealed too. The net time allotted to the increments must be reduced because more time is seen to be apportioned to the hiatuses as the resolution increases; but the net thickness does not change; so the accumulation rate of the depositional increments must be found to increase as their individual thicknesses decrease. Thus, the mean (or expected) rate of sediment accumulation becomes inversely related to time span [2,4]. The empirical relationship between accumulation rate and time scale will be shown to be a “negative power law” - this term conveniently describes an inverse linear relationship between the logarithms of two variables. Such relationships characterize fractals [5].

**Table 1:** Properties of Different Representations of Sediment Accumulation

	ACCUMULATION HISTORY (FIG. 1a)	STAIRCASE PLOT (FIG. 1b)	TIME LINES (FIGS. 1d, 1e)
Appearance	Time series with positive and negative slopes	Time series with only zero and positive slopes	Broken lines
Represents	Complete history of the deposit thickness during accumulation process	Age and elevation of horizons in preserved sedimentary section	Portions of elapsed time recorded by deposition (1d) or by preserved sediment (1e)
Metric space	Thickness - Age	Elevation - Age	Age
Shortcoming	Practical, but not explicit, lower limit of age resolution		
Range of fractal dimension	1.0 - 2.0	1.0 [19]	0.0 - 1.0
Possible fractal analogs	Fractional Brownian walks	Devils Staircase	Cantor Bar, Levy Dust
Physical significance of fractal dimension	Degree and nature of feed-back during accumulation	A borderline fractal! [19]	Completeness of stratigraphic record (Fig. 1e) or continuity of deposition (Fig. 1d)

### Graphical Representation of Sediment Accumulation

The most common representation of sedimentary deposits is a columnar section, drawn against a thickness scale (Fig. 1c). To appreciate the fractal nature of sediment accumulation and the role of hiatuses, however, we need a graphic that includes a time scale. Six options are considered below. In the first four, time is represented by a scale of age (Table 1). Two of these options (accumulation history and staircase plot) retain the thickness scale, so that the slope of the graph shows accumulation rate. The two time-line options do not; they record only the presence or absence of sediment for a given time interval (Fig. 1d, 1e). Notice (Fig. 1) that none of these options can be drafted in practice without an implicit and artificial lower limit of time-resolution. The fifth and sixth options make time-resolution explicit; they plot the durations of time intervals (i.e. resolution), rather than their ages. Although this ploy loses all information about the sequence of the intervals, it allows empirical data from many different deposits to be combined seamlessly into one graph.

**The Accumulation History:** If all the successive elevations of the sediment surface are plotted in historic sequence, the resultant time series represents a complete accumulation history (Fig. 1a). It includes details of all the depositional and erosional episodes during time spans that are ultimately recorded by surfaces of hiatus. Notice that those portions of the accumulation history that represent hiatuses begin and end at the same elevation. If erosion has occurred, these portions assume an asymmetric form: they begin with positive slope in a depositional interval, arch over a maximum,

and end with zero slope at minimum. These portions of the plot would not be recoverable from real stratigraphic sections even if every preserved horizon were dated.

**The Staircase Plot:** A complete accounting of the elevation/age coordinates for all *preserved* horizons in a sedimentary deposit would assume the form of an irregular staircase (Fig. 1b) in which hiatuses appear as horizontal treads. The staircase plot omits those segments of the accumulation history that have negative slope (erosion episodes) or are higher than subsequent minima (erosional losses of sediment). Reconstruction of the staircase plot from real sections is limited only by the number of datable horizons. Notice that the accumulation history lies *on or above* the staircase plot, never below it. The recoverable staircase plot provides a lower bound for the true accumulation history; thus, the staircase plot is a systematically biased estimate of the accumulation history.

**Two Time-Line Plots:** Sediment accumulation may also be represented as one-dimensional time lines (Fig. 1d, 1e) that ignore the elevation of the sediment surface. The solid portions of the time line report that the corresponding time interval is characterized by some deposition, regardless of thickness. Two versions of the time line are possible. One reports only the intervals of deposition that survive subsequent erosion (Fig. 1e) and thus represents the *completeness of a stratigraphic section*. The breaks in this line mark the duration of hiatuses. Alternatively, all depositional intervals may be included (Fig. 1d), whether later lost to erosion or not. In this case, the time line records the *continuity of deposition*, i.e. what an observer would have noted at the site of deposition. Line 1e is a one-dimensional projection of the risers in the staircase plot; line 1d is the corresponding one-dimensional projection of the accumulation history.

**Plots of Rate or Thickness against Time Span:** The previous four diagrams are useful images for single sedimentary sections. But they are difficult to reconstruct in detail because real deposits typically include too few dated horizons. Neither do they lend themselves readily to combining data from more than one stratigraphic section onto a single line -- the demands upon time correlation are typically too hard to meet. If thickness or rate are plotted against the duration of the dated interval rather than its age, however, measurements from many different deposits or stratigraphic sections can be combined into one graph.

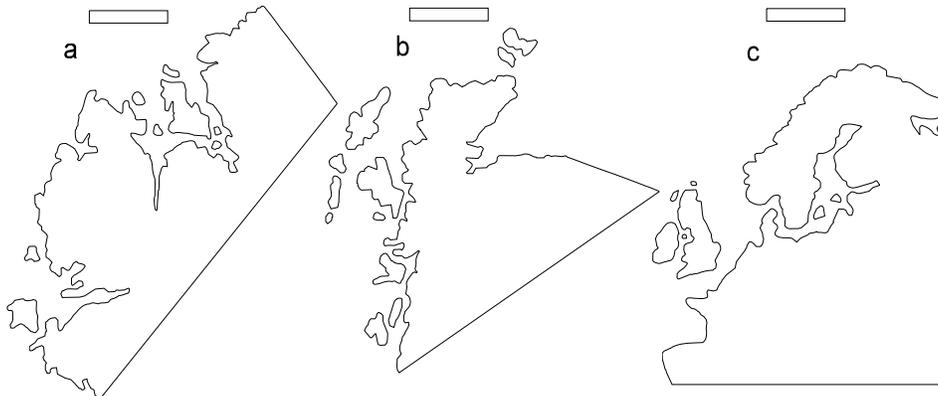
Logarithmic plots of average accumulation rate against time-span summarize nearly all of the empirical data that support later sections of this paper (Figs. 4-6). The logarithmic scales effectively display the wide range of measured values and reveal the negative power law relationships. But the graphs may initially trouble some readers, especially those unfamiliar with fractal geometry, because they plot a fraction (thickness/time) against its denominator (time). This practice deliberately highlights the scale-dependence of rates. In attempting to prevent application of rates to the wrong time spans, however, it invites an improper interpretation of correlation coefficients. It is crucial to realize that contours of constant thickness can always be drawn on a plot of rate against time. On a logarithmic graph, these contours are straight lines with a slope of -1.

On a plot of accumulated *thickness* (y-axis) against time span (x-axis), a regression with zero slope would imply that accumulated thickness is independent of time span; the correlation coefficient would be zero. Plotted as accumulation *rate* against time span, however, the same data would generate a -1 slope and a non-zero correlation coefficient. Thus, some readers may worry that regressions of rate against time span include a spurious component of correlation. The degree of correlation is spurious if attributed to thickness and time. Some negative correlation of accumulation rate and time span is indeed a mathematical inevitability whenever the deposit includes hiatuses -- the highest rates can be sustained only for relatively short time intervals. The peculiarity of the plot and the trouble with accumulation rates are one and the same. Unfortunately, more geologists tend to worry about the plots than to worry about using rates without regard to time scale. Concern about improper use of correlation coefficients should translate directly to concern about improper use of accumulation rates. Notice also that if thickness were truly independent of time span (power law gradient of -1 on these plots) this would be a highly significant observation for

stratigraphy. It would imply that the thickness of deposits generally provides no guide whatever to the amount of time that they span. Such steep negative slopes are, in fact, quite rare in the empirical data and persist for limited ranges of time span. We shall see that they can be explained in terms of periodic components in the accumulation history.

For steady accumulation without hiatuses, rate would be a constant and independent of time scale; the plot of rate against time span would assume a zero slope if accumulation were steady. Positive slopes should never result from an accurate and complete set of measurements. Negative slopes indicate the presence of hiatuses and an incomplete stratigraphic record [2]. Steeper negative slopes reflect increasing incompleteness. The limiting steepness is  $-1$  -- constant expected thickness regardless of time span. Only an unrealistically stylized cyclic accumulation history could reach this limit. It would have to be imagined to operate as follows: the depositional process must generate rapid periodic alternations of depositional and erosional phases in which all depositional increments have the same thickness and are precisely eroded away before the next increment is added. Regardless of time span, all measurements of accumulation rate would then capture no more than one sediment increment. There is no net accumulation in this model at time spans longer than the alternating phases; it is the limiting extreme case of stratigraphic incompleteness. The significance of slopes between 0 and  $-1$  will be interpreted in a later section from some basic equations of fractal mathematics.

Our crucial lesson from fractal mathematics will be that single measurements of a property that is dependent upon time scale may have no validity beyond the time span of measurement. The lesson is generally better appreciated in the context of length scales and coastlines. And so, for readers who are not familiar with fractals, the next section reviews the role of spatial scale and the hierarchy of embayments in the problem of coastline length. A firm foundation in purely spatial patterns eases the conceptual leap to the fractal view of time series, which is at the heart of understanding the influence of time scale and hiatuses on accumulation rate. Of course, the time scale introduces significant differences between coastlines and accumulation histories which should be understood as well as the analogies between coastline length and accumulation rate (Table 2). The analogy is intended to provide a more familiar example of a property that is in part a function of the scale of measurement.



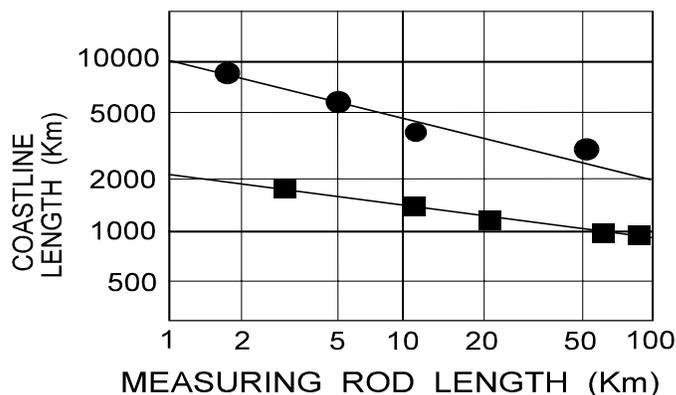
**Figure 2.** Coastlines drawn at different scales. Details of the coast of Lewis (a: 10km scale bar), which is an island on the coastline of northern Great Britain (b: 100 km scale bar), which is an island on the coastline of northwest Europe (c: 1000km scale bar). All three maps have comparable complexity; nothing inherent in the pattern of the coastline would reveal the scale of the maps to a stranger.

### The Fractal View of Coastlines

It is simply not reasonable to seek a single value to describe the length of a coastline. Unless the coast is uncommonly smooth, it has no unique length [6,7]; the mapped length of a coastline varies

with map scale (Fig. 2). Introductions to fractal geometry routinely explain this coastline paradox. For reasons that are closely related, but only rarely explained [8,9], no single value may reasonably be expected to describe the rate of accumulation of a sedimentary deposit. Mean measured accumulation rates vary with the degree of resolution achieved within the hierarchy of sediment increments and hiatuses.

Given a map, measurement of the length of a coastline presents no great difficulty. Yet the value obtained has very little meaning: if the map scale had been different, the measured length of the coastline would probably have been different too. A larger scale generally reveals more detail in the hierarchy of embayments (Fig. 2), leading to a greater measured length. The length of a coastline is evidently a property of both the coastline and the scale of measurement. The scaling relationship is a negative power law (Fig. 3).



**Figure 3.** Two examples of negative power laws that describe coastlines. The circles represent measurements from a more intricately complex coastline with a steeper negative regression and a larger fractal dimension than the coastline reported by the rectangles; its length changes more dramatically with map scale than the simpler coastline represented by the rectangles.

The slope of the power law describes the complexity of the coastline. As the shape of the coastline becomes more intricate, the negative slope steepens. A uniform slope means that the coastal complexity appears the same across all scales. The bays and headlands are self-similar at all scales and only somebody familiar with the region can recognize the map scale from the form of the coastline alone. If the slope of the power law changes at some spatial scale, then the complexity of the coastline changes as a function of scale; in other words, bays are not equally well developed at all scales.

More intricately complex coastlines are said to have a larger fractal dimension. Fractal dimension is a scale-independent measure of how densely the spatial fractal occupies the space within which it lies. For example, a set of points arrayed to form a straight but broken line is a pattern intermediate between a single point (dimension = 0) and a complete line (dimension = 1). The term fractal describes this concept of “fractional” dimensions. A coastline occupies part of a two dimensional map space; so its fractal dimension may range between 1 and 2. The map of a perfectly smooth straight coast would be a Euclidean line. Its fractal dimension would be 1.0. The decimal form is used deliberately to admit the possibility of dimensions with fractional values. More intricate coastlines effectively occupy more of the map. In the limiting case, which is unattainable for a real coastline, a whole area of the map would be filled by a line that keeps doubling back upon itself.

**Table 2:** Comparison of Fractal Properties

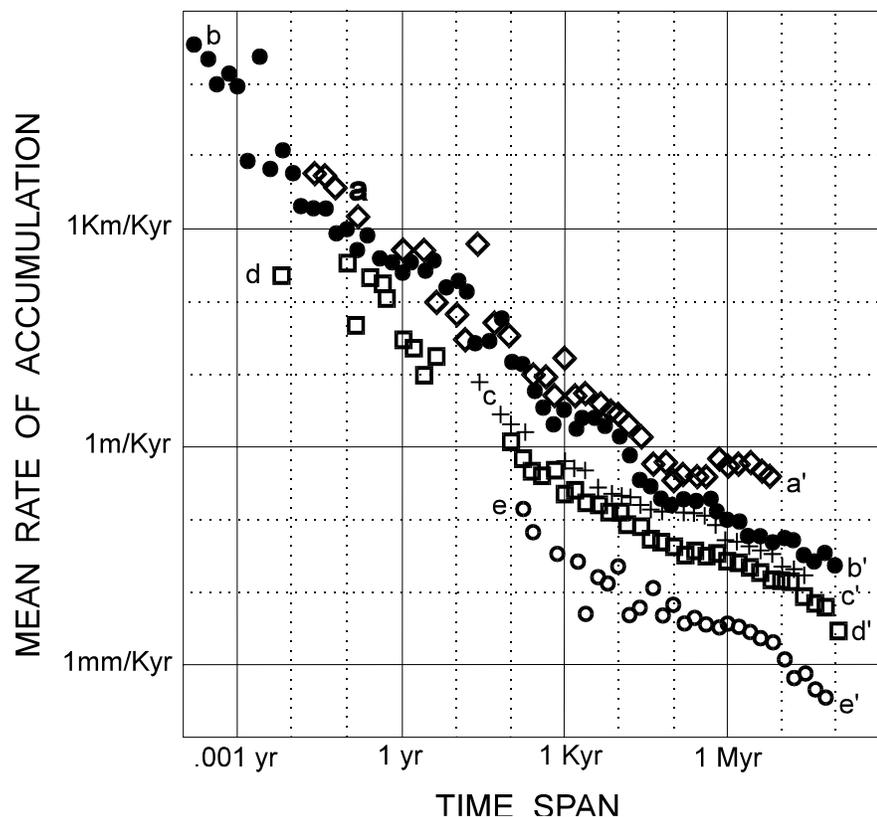
PARAMETER:	LENGTH OF COASTLINE	RATE OF SEDIMENT ACCUMULATION
Coordinate system	Latitude and longitude (two length scales)	Elevation or thickness and time (a length scale and a time scale)
Simple formulation	Number of measuring rods, laid end-to-end, that mimic coastline (or number of map cells which coastline enters)	Net thickness of deposited sediment divided by the time elapsed between onset and cessation of deposition
Pitfalls	Single values should not be extrapolated or compared without attention to length scale	Single values should not be extrapolated or compared without attention to time scale
Source of fractal complexity	Nested hierarchy of bays, headlands and islands	Nested hierarchy of sediment increments and surfaces of hiatuses
Critical scale	Length of measuring rod (or size of counting box)	Time span over which rate is averaged
Diagnostic regression	Log.-measured-length as a function of log.-size of the measuring device	Log.-rate as a function of log.-time-span of measurement
Range of possible gradients	0.00 to -1.00 A smooth straight coast would generate 0.00	0.00 to -1.00 Steady accumulation generates 0.00 A random walk generates -0.50
Corresponding fractal dimension	1.00 to 2.00	0.00 to 1.00 for the completeness of the sedimentary section (a broken line) Between 1.0 and 2.0 for the accumulation history (a time series)
Meaning of steeper negative gradient	More intricate and complex pattern of embayments	Less complete stratigraphic record; i.e. more time recorded by hiatuses
Lower limit of determination	Surface texture of cliff materials or beach particles (length becomes impractically large)	Instantaneous arrival of individual grains at sediment/fluid interface (rate becomes infinitely large as time span approaches zero)
Upper limit of determination:	Size of the Continent	Overall duration of the sedimentary deposit or stratigraphic section
Significant differences	<ol style="list-style-type: none"> <li>1. Coastline may intersect same longitude at different values of latitude and vice versa.</li> <li>2. The nested bay-headland motif may be self-similar.</li> <li>3. Coastlines are a sets of continuous loops (continents and islands).</li> </ol>	<ol style="list-style-type: none"> <li>1. Sediment surface may have the same elevation at different times, but only one elevation for each moment in time.</li> <li>2. The nested hiatus-increment motif may be self-affine.</li> <li>3. If single grain arrivals are resolved, then accumulation histories are seen to have discontinuities in elevation.</li> </ol>

### Sediment Accumulation as a Fractal

Just as the measured length of a coastline requires a statement of the distance scale before it has real meaning, so too the measured rate of sediment accumulation or an estimate of the completeness of a stratigraphic section should be accompanied by a statement of the time scale. In other words, the rate of accumulation is a property of both the depositional system and the time scale. The diagrams in Figure 1 all have an arbitrary limit on time resolution. If redrawn with finer resolution, the time lines (Fig. 1d, 1e) include more breaks and the staircase plot includes more steps. Notice that the inclusion of more treads (in the magnified images) forces the steps to become shorter *and steeper* because the long-term slope is fixed. The incremental accumulation rates inevitably increase when examined at finer resolution.

The broken time lines that represent the completeness of a stratigraphic section may obviously be considered as fractals with a dimension between zero and one. Readers familiar with fractal

geometry may be reminded of the “Cantor Set” fractal [20]. A Cantor Set is a string of dashes (Cantor Bar) or points (Cantor Dust) generated by erasing part of a straight line, and then repeating the erasure pattern on the shorter line segments that remain. We might use the Cantor Bar as an analog for stratigraphic time lines, but the rules for preparing the bar hardly compare with real depositional processes. Furthermore, the Cantor Bar generates a wide range of hiatus lengths, whose ages always reveal a nested regularity. When a random distribution of hiatuses is preferred, then the fractal mathematics of the Levy Dust are appropriate [21]. The Cantor Set is intimately related to another fractal, the Devil’s Staircase [19], which could be used to model the stratigraphic staircase plots. The Devil’s Staircase has repeating and nested patterns in its sequence of steps; stratigraphic staircase plots should be free to exhibit random step distributions.

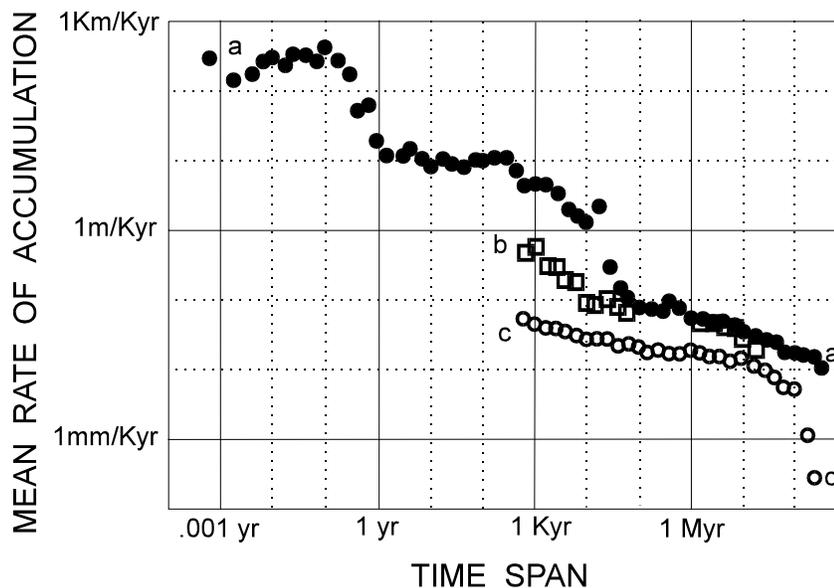


**Figure 4.** Mean accumulation rates for terrigenous sediments on passive continental margins. a-a’: deltas (diamonds; 2,988 empirical rate determinations); b-b’: shelf seas (filled circles; 22,636); c-c’: continental slopes (crosses; 6,421); d-d’: continental rises and abyssal plains (squares; 10,821); e-e’: abyssal red clays (open circles; 2,215). Rates are averaged for logarithmically scaled windows of time span; there are five, non-overlapping windows for each order of magnitude.

Although the Cantor Set and Devil’s Staircase produce patterns that could simulate some graphical representations of sediment accumulation, it is important to realize that the algorithms used to generate these fractal images are not necessarily analogous to any sedimentary stratigraphic process. These fractals are two dimensional spatial images and some simple operations in space make no sense in the time dimension because time has a real irreversible sense of direction. Of course, mere comparison of images is the least sophisticated application of fractals. Fortunately, a large body of fractal mathematics concerns time series, rather than spatial patterns, and is these fractal relationships that are directly applicable to understanding accumulation rates.

The accumulation history of a sedimentary deposit is a time series that must incorporate the fluctuations in the level of the sediment surface which represent a nested hierarchy of hiatuses. By analogy with coastlines, we might expect the complexity of the accumulation history to be described by a fractional dimension between 1.0 and 2.0. But the coordinates of each point are now elevation (or thickness) and time. Technically, alteration of one axis from a length-scale to a time-scale has moved us from self-similar fractals to the realm of self-affine fractals. The term self-affine acknowledges that the elevation and time coordinates cannot be measured in the same units. There is no intrinsically correct proportion for the vertical and horizontal scales of an accumulation history.

Experts differ on the propriety of assigning a fractal dimension to self-affine fractals. Instead, there are named fractal coefficients that assume values in a fixed range to describe the relative complexity of time series. These coefficients are analogous to the fractal dimension of self-similar fractals. Both self-similar and self-affine fractal scaling are described by power laws and the slopes of the power laws relate directly to the fractal dimensions and coefficients respectively. The best proof that sediment accumulation has fractal properties is a power-law dependence of accumulation rate upon time span.



**Figure 5.** Mean accumulation rates for marine carbonate sediments. a-a': peritidal platform and reef deposits (filled circles; 27,029 empirically determined rates); b: periplatform apron deposits (open squares; 7,693); c-c': calcareous oozes remote from carbonate platforms (open circles: 68,315).

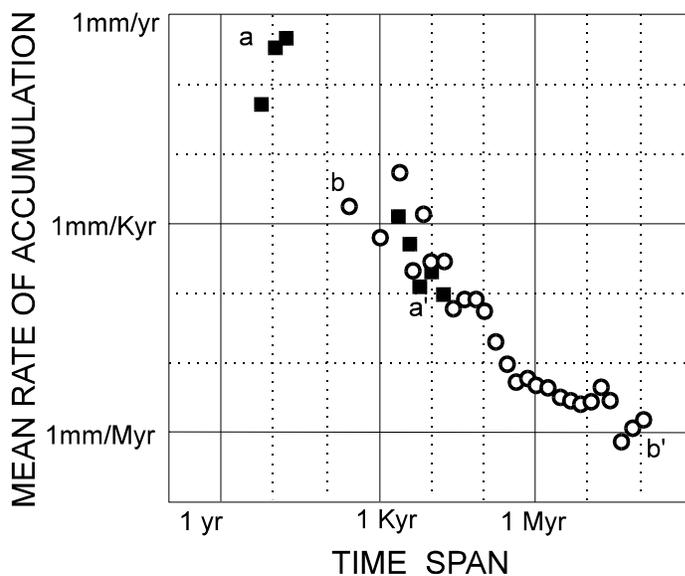
### Power Law Dependence of Accumulation Rate upon Time Span

The empirical data for this paper are large compilations of accumulation rates and their dependence upon the time span of measurement. Plots of rate against time span have been prepared for a wide range of depositional environments (Figs, 4-6) [2,4,10]. For the length of a particular coastline, a few measurements at different scales suffice to reveal that the scaling follows a power law. To characterize the expected rates of accumulation for a whole depositional environment considerably larger samples are needed. A small number of rate determinations may produce a quite spurious slope in this exercise, because the range of measurable rates at any one time scale is quite large. The empirical relationships in figures 4 and 5 stabilized after the compilation of thousands of rate determinations from many different sites.

Although each time span is characterized by a large variance in the measured rates, this does not mask the overall negative regression for large samples. Notice that the negative gradients characterize a very wide range of time scales; we must reckon that the sedimentary stratigraphic record incorporates hiatuses with this same wide range of durations. The steepness of the negative slope and, thus, the expected incompleteness of stratigraphic sections varies with environment of deposition, especially at short time spans. But differences that may be attributable to environment of deposition decrease with increasing time span [2]. This probably means that depositional processes are not the primary influence on the distribution of hiatuses at time scales longer than about ten thousand years. At long time scales the most influential factor must be less dependent upon depositional environment and operate on a larger spatial scale to explain the convergence of the power laws. Lithospheric subsidence likely becomes the most significant determinant of the long-term accommodation of sediment. It can explain the long-term convergence of plots from different shallow-marine environments. Accumulation on abyssal ocean floors would not be limited by subsidence; and so, the difference between abyssal and shallow-marine accumulation rates can be sustained to very long time spans.

### The Problem of “Representative” Accumulation Rates

It is evident from figures 4-6 that far more of the total variability in accumulation rate is explained by the time span over which the rate is determined than by either the depositional environment or the age of the sediments. The characteristic accumulation rates for one depositional environment may be represented by a single power law that describes how rate scales with time span, but not by a single rate. Although it is reasonable to seek a single accumulation rate that represents the average for a depositional environment *at a given time scale*, even this exercise causes trouble.



**Figure 6.** Mean rates of accretion of marine manganese nodules and crusts from shelf seas (squares a-a', 85 determinations) and abyssal depths (circles b-b', 1,054).

A random, short term observation of most modern flood plains and shelf seas, for example, would most likely be unable to measure any sediment accumulation. Much of the time, sediment does not accumulate. Only a very long series of short-term observations would ensure that the mean rate included a representative number of zero values. And in practice it would be impossible to know when the series was long enough. Empirical logarithmic plots like figures 4-6 seem, at first glance, to compound the problem because they cannot show negative or zero values anyway. They show the average accumulation rate only for intervals in which there is some *measurable net deposition*. The proper proportion of intervals of zero net-deposition, which is so difficult to measure directly, is now

recoverable, however; it is revealed by the reduction in mean rate at longer time spans. If the average rate for ten thousand-year intervals is four times that for hundred thousand-year intervals, for example, then only one in four of the ten thousand year intervals need to be represented by sediment increments, in the longer term, and the other three fourths must be hiatuses.

In this way the steepness and length of the negative slope of any segment of the plot describes the incompleteness of the sedimentary record at the time scales of its end points [2,11]. The ratio of the long-term to the short-term accumulation rate is the proportion of elapsed time represented, on average, by sediment when a stratigraphic section at the long time scale is resolved into sediment increments at the scale of the short term end point. The ratio decreases as the length and steepness of the segment increases. This quantifies our expectation that the stratigraphic record must reveal more and more hiatuses when viewed at finer and finer resolution.

Because the empirical scaling laws in figures 4-6 are based upon rates from many different locations, purely local differences in the frequency distribution of hiatuses are likely to cancel one another out and have no impact on the general slope. The resulting negative power laws capture only the globally persistent features of the hierarchy of hiatuses. They may reflect the characteristic time scales of processes that accumulate and accommodate sediment. In order of increasing time scale, these processes would be the depositional mechanisms, global sea-level and climate change, and lithospheric subsidence. Tides and storms may dominate shallow marine accumulation at time scales of a few years down to a few hours. At time scales of tens and hundreds of thousands of years, astronomically forced climate change is likely to become a more powerful influence of sediment accommodation. Cooling and subsidence of the oceanic lithosphere accommodates stacks of eustatic cyclothem at time scales of tens of millions of years.

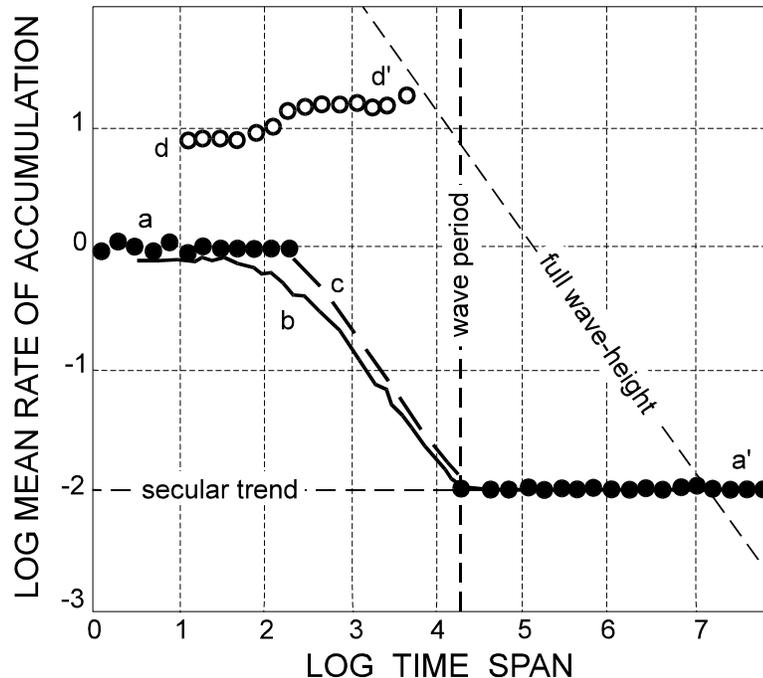
In shallow marine environments (Figs. 4ab, 5a) the negative slopes tend to steepen markedly at time scales of tens of thousands to hundreds of thousands of years. The simplest explanation [11] is that hiatuses with this periodicity are more prevalent than hiatuses at longer or shorter intervals. Thus, the negative power laws seem to capture the impact of Milankovitch scale rhythms in eustasy and sediment accommodation. Because these cycles are driven by global climate change, it is reasonable to suppose that their influence upon accumulation rates would survive the process of compositing measurements from hundreds of different localities. For carbonate platforms (Fig. 5a), average accumulation rates fall to 10-20% of their shorter term values over the Milankovitch range of time scales. This implies that the surfaces of hiatus, which many stratigraphers use to delimit meter-scale "cyclothem," record 80-90% of the time in each accommodation cycle [11]. Thus, interpretation of the slope of the power law has answered the difficult question of the proper apportionment of intervals with no net deposition.

As the time span tends to zero, the negative scaling laws in figures 4-6 imply that accumulation rate could increase without bound. As we have already seen, this is an entirely realistic expectation because sediment is deposited as a succession of discrete particles. At the moment of arrival of a single sediment grain, accumulation rate is infinitely rapid; the entire finite grain thickness instantaneously becomes part of the deposit.

### **Interpreting the Slope of the Power Law**

Numerical models can reveal many aspects of the significance of the slope of the power laws. The models allow the accumulation histories to be exactly prescribed; they facilitate much more intense sampling of age and elevation coordinates than is possible in real stratigraphic sections; and they eliminate the measurement error associated with empirical data. For some fractals that can mimic key properties of stratigraphic accumulation histories, staircase plots and time lines, there are standard equations from which the form of the negative power law may be determined directly. For other numerical models of sediment accumulation, however, plots of rate against time span can more easily be prepared from the outcomes of numerous computer runs.

Both approaches will admittedly investigate much simpler time series than the accumulation histories of real sedimentary deposits; they are models rather than simulations. The goal of the exercise is to produce time series that isolate different aspects sediment accumulation histories with regard to the distribution of hiatuses and to look for corresponding differences in the power laws. By manipulating the various components of time series it is hoped to recognize the contribution of each to the power law. These components are the steady trend or 'drift' (which might model long term subsidence), periodic components (climate and sea-level cycles, perhaps), and the stochastic component (some aspects of the depositional process). We shall start with a simple periodic model and then examine the Cantor Set analogy. The stochastic element will be modeled as a Brownian Walk, which is a fractal time series with standard equations that can be adopted for geologic situations.



**Figure 7.** Mean accumulation rates for a sine-wave model of periodic accumulation. The model adds a single sine wave (period and wave-height shown) to a secular trend (constant positive slope). The wave-height plots as a line with slope -1 because it is a line of constant thickness. Mean rates are illustrated as determined from: a-a': dated rocks in cycles that have completed their erosional phases; b: dated changes; and c: dated hiatuses; d-d': dated rocks from the youngest cycle, prior to any erosional losses.

**Regular Periodic Models of Accumulation:** To isolate the influence of a regular periodicity in the distribution of hiatuses, eliminate all stochastic components from the model and leave just one sine wave and the trend component [2]. The model generates hiatuses with fixed size and spacing. The resulting regressions of rate upon time span (Fig. 7) have three segments with different slope. In the longest- and shortest-term segments, the slope approaches zero. Measurements at time spans much longer than the interval between hiatuses are insensitive to the shorter and regularly spaced hiatuses; they see only the steady trend. At time spans much shorter than the duration of the hiatuses, measured rates capture only the rates of accumulation within the sediment increments. Between these two time scales, the slope is steeper, approaching -1.0. The steep portion approximates a line of constant thickness equal to the thickness of the individual sedimentary cyclothems that remain after completion of the erosional phase of the cycle.

The break in slope at the shorter-term end of the steep segment approximates the duration and thickness of the sediment increments between hiatuses. The height of the steep segment increases

with the fraction of the cycle that is recorded by hiatus. Thus, the pronounced step in the empirical scaling law for accumulation rates in shallow carbonate environments (Fig. 5a) was explained in terms of a pervasive periodic control upon sediment accumulation -- Milankovitch-scale eustatic cycles [11]. For shallow-marine terrigenous sediments, the corresponding step is less distinct (Fig. 4b); perhaps the difference is attributable to fluvial processes that continue to distribute sediment on sandy shelves during low-stands of sea level.

If more sine waves with different period and amplitude are added to the periodic model, the scaling law must incorporate more steps [2,11]; the steps tend to merge and interfere with one another, giving the impression of a less steep overall slope. Almost any accumulation history could be simulated by including a large enough number of different periodic components with carefully selected period and amplitude; this principle underlies Fourier analysis. Eventually, however, plots with multiple periodic components become indistinguishable from those generated by models which are dominated by the stochastic component. Although the numerical model has revealed the influence of a dominant periodic component on the negative power laws, the exercise should not be compounded in an attempt to reach detailed simulation of the empirical plots unless it is realized that the solutions will not necessarily represent the real depositional dynamics.

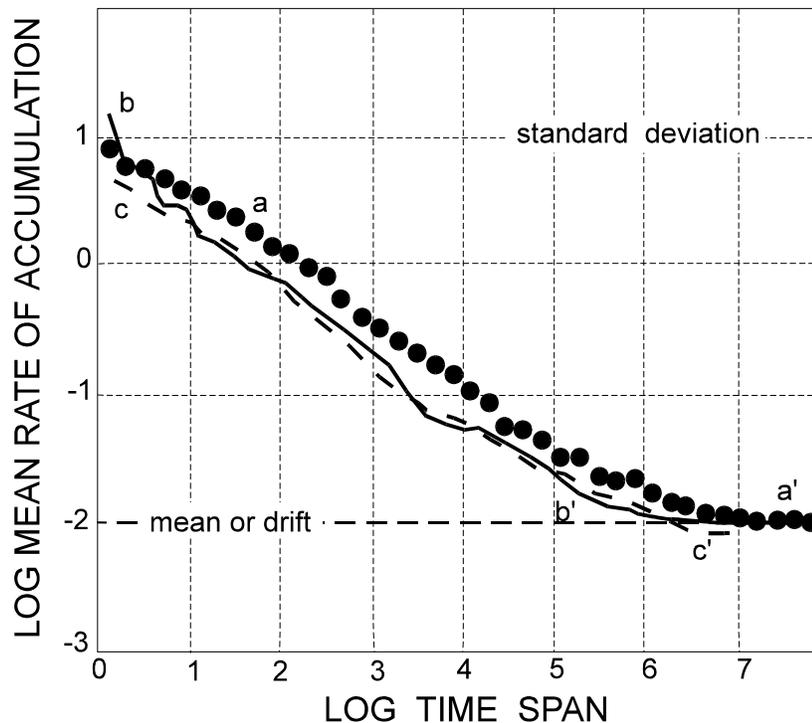
**The Cantor Bar Analogy:** It is instructive to examine briefly a standard result for the Cantor Bar fractal because it reveals how simply the slope of the negative power law can be related to fractal dimension or fractal coefficient. Like a stratigraphic time-line, the Cantor Bar does not model thickness. We must recast its descriptive equations into the language of sedimentary stratigraphy by including a constant,  $K$ , that depends upon the overall rate of accumulation for the whole section [9,21]. The expected rate of deposition,  $r$ , predicted by the Cantor model for a time interval of length  $t$  is then a decreasing power law function of that time interval:

$$r = Kt^{(d-1)} \quad (1)$$

which has a slope  $(d-1)$  in logarithmic coordinates:

$$\log r = (d-1)\log t + \log K \quad (2)$$

Equation 2 has the standard form for a straight line in logarithmic coordinates ( $y = mx + C$ ; where  $m$  is the slope of the line). The exponent  $d$  must vary between zero and one so that  $d-1$  can model empirical plots with slopes between zero and minus one. Plotnick [21: Eq. 1.9] equates  $d$  with the "Lipschitz-Holder exponent" of fractal mathematics. It is the fractal dimension of the Cantor Bar [9: Eq. 2.13]. So, the fractal dimension of the time-line graphics (Fig. 1d-e), which describe the continuity and completeness of accumulation, will be one plus the slope of the power law that relates accumulation rate to time span. The completeness of the time-lines falls as we examine them at finer and finer resolution. The rate of loss of completeness with increasing resolution is determined by the fractal dimension; and the dimension can be calculated from the slope of the negative power law as revealed by empirical determinations. In Figures 4 and 5 the slopes tend to be somewhat less steep, and the expected completeness correspondingly higher, for deeper marine deposits and at long time scales than in short-term accumulation above wave base.



**Figure 8.** Mean accumulation rates for a Brownian motion model of episodic accumulation. The individual erosional and deposition increments are normally distributed with mean and standard deviation as shown. Mean rates are illustrated as determined from dated rocks (circles a-a'), dated changes (line b-b') and dated hiatuses (dashed line c-c'). For an explanation of these three methods, refer to the text.

**Purely Random Models:** In the simplest purely stochastic models, each increment of deposition or erosion is independent. The outcome in one time interval provides no information to improve the prediction of what will happen in the next. Sadler and Strauss [4] describe a very simple coin-tossing model that meets this criterion and can simulate surprisingly many aspects of accumulation rates: on heads, deposit one increment; on tails, erode one increment. But more insight results from modeling sediment accumulation as a one-dimensional Brownian walk [22] because these walks have been explored at length in fractal mathematics.

To model sediment accumulation as a Brownian walk, represent the time series of net sediment accumulation by the sum of a succession of tiny independent increments of deposition or erosion, selected at random from a normally distributed set of thicknesses. When the mean of the distribution is zero, the model has a stochastic component but no trend: one half of the distribution generates increments of deposition; the other half generates erosion; and selections from the two halves tend to balance exactly in a long run of random selections. The succession of increments is called a “white noise” to distinguish it from the “walk” which is the history of their running total. Setting the mean greater than zero ensures long-term net accumulation, because more than half of the distribution will generate deposition; this is equivalent to adding a positive trend component to each increment. A Brownian walk without a trend component generates a negative power law with a slope of -0.5. Inclusion of a positive trend (Fig. 8) causes the negative slope to decrease at long time spans and approach the steady rate (zero slope) of the trend component.

We can use the relationship between fractal dimension and the slope of the power law developed for the Cantor Bar (Eq. 2). It implies that a stratigraphic time line that represents those sediment increments which escape erosion in the Brownian model will have a fractal dimension ( $d$ ) of 0.5. The value of one half reflects the fact that the divergence of a Brownian walk from its starting

elevation scales with the square root of its duration. In a more general formulation, the exponent  $H$  describes how the elevation change ( $\Delta h$ ) between two points on a self-affine fractal walk scales with  $t$ , their separation in time [19: sect. 9.5; 9: sect. 7.3]:

$$\Delta h = Kt^H. \quad (3)$$

Divide both sides by  $t$  to obtain the scaling law for slope or rate:

$$r = Kt^{(H-1)}. \quad (4)$$

The  $H$  exponent of a Brownian walk is 0.5. The fractal dimension ( $D$ ) of the walk itself must be greater than the dimension of the broken time line it generates; it is a complex curve with a dimension between 1.00 and 2.00. The value of  $D$  for Brownian motion is 1.5. The sum of  $H$  and  $D$  is 2.0 for any fractal time series. This standard relationship that may be reorganized (Eq. 5) and combined with equation 4. The result (Eq. 6) shows that the gradient of the power law which relates rate to time span must be  $1-D$  for the accumulation history:

$$(H - 1) = (1 - D) \quad (5)$$

and

$$r = Kt^{(1-D)}. \quad (6)$$

The reader will easily discover many more basic properties of random walks in the mathematical literature. But we must take care to extract the geologically appropriate ones. For example, mathematicians convert a Brownian walk ( $D = 1.5$ ) to a Levy dust ( $d = 0.5$ ) by considering only the points at which the stochastic component of the walk would intersect a fixed elevation value. This has the appearance of deriving a time-line from an accumulation history; but we need a different “dust” -- the one created by considering all the points that are ultimately recorded by preserved sediment (Fig. 3e) [4]. Strauss and Sadler [23] solved this problem numerically. Because the mathematics can be quite daunting, figure 8 was not prepared from their equations but by extracting hundreds of thousands of measurements from repeated computer simulations.

**Random Models with Memory:** Textbooks on fractals also discuss “fractional” Brownian motions [19: sect. 9.5; 21: sect. 1.9; 9: sect. 7.2). Such walks possess a degree of positive or negative correlation between increments. This property is more colloquially called feed-back or memory. It means that the accumulation history in any one time interval provides some information with which to improve the guess about what will happen in the next interval. This has obvious appeal for models of sediment accumulation. At some time scales, depositional features like channels, levees, fan lobes, and reefs tend to be stationary and encourage the persistence of accumulation in the same place. At longer time scales the very build-up of sediment encourages a shift to new locations, a negative persistence. With knowledge of the prior history of accumulation, we often can make better guesses about future changes; accumulation is typically not a purely random process.

Positive correlation (persistence) reinforces change and causes the time series to diverge from its initial elevation faster than Brownian motion; that is, the  $H$  exponent will be greater than 0.5 and the dimension  $D$  will be correspondingly less than 1.5. Negative correlation has a damping effect upon divergence and implies  $H$  less than 0.5 and  $D$  greater than 1.5. It is possible to run computer simulations with this wider range of  $H$  values and examine the resultant plots of rate and time span. The programming is more complex than for Brownian walks and the exercise is really not necessary. Enough relationships have already been derived between the fractal dimension of the stratigraphic time line ( $d$ ), the  $H$  exponent of the time series, and the slope of the negative power law to allow the results to be anticipated. Models with positive correlation must generate power law slopes less steep than -0.5. In the extreme case, correlation is perfect, accumulation is steady and the slope of the power law  $(1-D)$  is zero ( $D = 1.0$  and  $H = 1.0$ ). Negative feedback leads to steeper negative slopes. In the extreme case of perfect negative correlation, every increment of deposition is subsequently exactly eroded and a power law with a slope of -1.0 emerges.

It is worth sounding a warning at this point that the profusion of named patterns and exponents for fractals can be confusing. The relationships reported here for  $H$ , can be found with  $H$  represented as

the Hurst exponent [19: sect 9.5] or the Hausdorff measure [9: sect. 7.1]. Probably because Hurst's exponent is more straightforward to calculate, it is a popular substitute for  $H$ . Turcotte [9: sect 7.6] relates the Hurst component as a property of the fractional noise to the Hausdorff measure as a property of the walk produced as a running sum of that noise. But the message of the previous paragraph remains straightforward: there is a very simple arithmetic relationship between the slope of the power law and the persistence of the accumulation process.

### **Failures to Appreciate the Role of Time Scale**

Two examples illustrate the pitfalls of ignoring the dependence of rate upon time span. The first concerns tectonism and terrigenous deposits. The second concerns the rate of accretion of chemical deposits in the deep seas.

Molnar and England have offered a physical explanation for the "illusion of accelerated uplift of mountain ranges" in Late Cenozoic times [12,13,14,15]. It had been popularly supposed that real tectonic uplift of the modern high plateaus and mountains, especially the Himalaya and Tibet, caused Quaternary climate change. Molnar and England caution that climate change could, independently, have caused accelerated denudation rates; the resulting changes in isostatic compensation would then quicken the creation of relief without real tectonic uplift. Much older changes in crustal thickness would be the ultimate cause of uplift. Their argument carefully distinguishes tectonic uplift from increasing surface elevation and respects the time scales of isostatic adjustment. But the fundamental time scale dependence of rate is ignored. For example, one argument they advance for accelerated denudation is the high rate of Quaternary sediment accumulation adjacent to high mountains, compared to earlier Cenozoic epochs.

Whether or not they are related, the Late Cenozoic accelerations claimed for accumulation rates and the creation of relief are both largely illusory because unequal time spans have been compared; and because it is reasonable to suppose that both accumulation and uplift are discontinuous. The Quaternary spans only 2 million years, the preceding Neogene 23 million years and the Paleogene 42 million years [16]. Even the individual epochs show a moderate positive association of duration and age. Thus, even if the pattern of unsteadiness is stationary, we must expect, from consideration of time span alone, that average Quaternary rates will exceed the Neogene rates which, in turn will be faster than the Paleogene values.

England and Molnar [15] are "unable to suggest a physical mechanism" to explain why rates scale with time span; quite so, it is a purely mathematical consequence of trying to describe an unsteady processes by its net rate of work. Proper analysis of the uplift of the Tibetan plateau certainly needs attention to the two-way interactions between climate and relief. It also necessary to remove the time-scale effects from the rate data [15].

Marine manganese nodules and crusts accumulate on deep ocean floors and beneath shallow shelf seas. Although it has been widely supposed that the deep marine nodules grow much more slowly than their shallow-water counterparts, the data do not unequivocally support this simple view [17]. Rates of growth determined from shallow water samples do exceed those typically calculated for abyssal samples by several orders of magnitude (Fig. 6). But, growth rates in shallow water have been determined for time spans that are generally much shorter than the minimum resolution of dated intervals in the abyssal nodules. Furthermore, deep water manganese nodules are known to accrete unsteadily: the concentrically zoned nodules include surfaces of hiatus and there is evidence of short term growth spurts that exceed the limits of radiometric rate determination [18]. The available data from all depths can be approximated by a single power law. This permits the view that growth rates in shallow marine nodules might closely resemble the short-term growth of the abyssal nodules and crusts. The growth of manganese nodules and crusts is certainly more persistent at abyssal depths than in shelf seas, it is not necessarily less rapid.

**Table 3** Properties of Dated Horizons as a Function of Dating Technique

	DATED DEPOSIT	DATED CHANGE	DATED HIATUS	DATED LEVEL
Examples	Radiometric dating; thermo-luminescence dating	Biostratigraphic zone boundary; paleo-magnetic reversal; isotopic stage boundary	Seismic sequence boundary	Surveying during active deposition
Dated horizon is in a depositional increment	Yes	Maybe	No	Maybe
Dated horizon is surface of hiatus	No	Maybe	Yes	Maybe
Dated horizon fixes point on accum. history	Yes; - ultimately preserved segments only	Sometimes	Unlikely	Yes; - both depositional and erosional parts
Dated horizon fixes point on staircase plot	Yes; - risers only	Yes; - risers or treads	Yes; - treads only	Maybe
Dated horizon fixes point on time line	Yes	Maybe	No	Maybe

### Dating Techniques

In the numerical models, the scaling law for accumulation rate has a simple geometric counterpart: the relationship between the slope and length of chords drawn between pairs of points on the time series. But we must take care to sample chord ends in a geologically realistic fashion. Because of erosion, many segments of the time series are not recorded by sediment in the final deposit. This section of the paper shows that the dating technique determines which parts of the time series can be sampled and, thus, can influence the form of the negative power law.

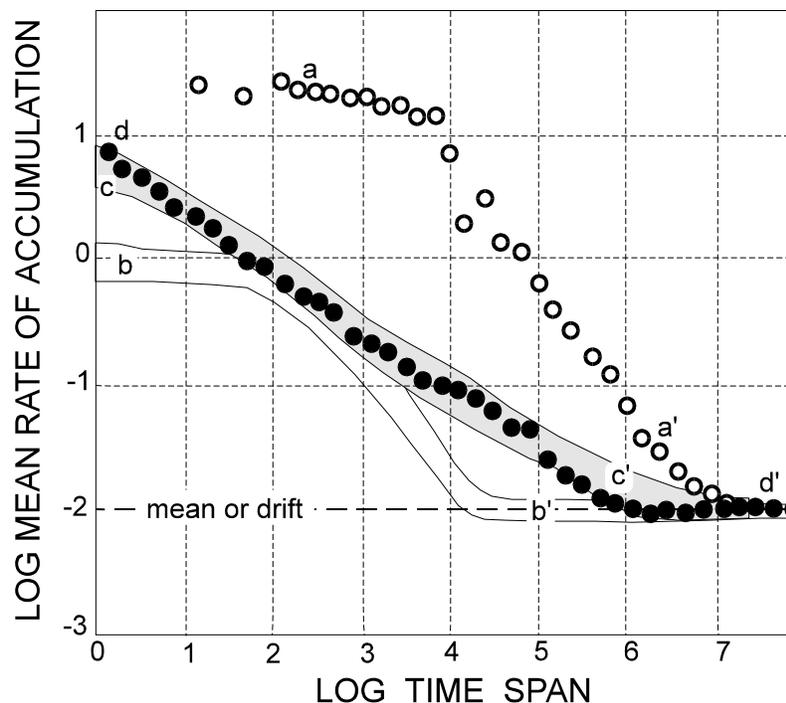
When a rate of accumulation is determined from the thickness of sediment between two horizons of known age in a sedimentary deposit, it will almost inevitably include some intervening intervals of hiatus. We now ask whether the known ages at the end of the dated interval might themselves fall within a hiatus. The answer varies with the dating technique. Dated deposits (e.g. radiometric dates), dated changes (e.g. magnetic polarity reversals), dated hiatuses (e.g. sequence boundaries), and dated levels (e.g. monitoring an active depositional system) behave differently in this regard (Table 3).

Radiometric methods can date deposits directly; the ends of an interval that is dated by two radiometrically age determinations must be anchored in depositional increments. Hiatuses may occur only within such a radiometrically dated interval, not at the ends. More often, however, rates are calculated between horizons that have been “dated” by recognizing changes in fauna, changes in magnetic polarity, or changes in isotopic composition for which the age has been calibrated elsewhere. These changes are recorded as a contrast between deposits “before-and-after” the event. The known age of the event is placed between two contrasting sediment samples or fossil faunas and may, therefore, fall within either a hiatus or a rock interval. Thus, biostratigraphic and magnetostratigraphic rate determinations potentially average across more hiatuses than radiometric determinations.

Sequence stratigraphic techniques are more extreme in this regard; they deliberately assign ages to surfaces believed to be hiatuses and use these surfaces to delimit correlative units. When a rate is

determined from the thickness of rock between two sequence-bounding surfaces of hiatus, and these surfaces are prominent on seismic cross sections, it is likely that both ends of the dated interval will lie within a significant hiatus.

The elevation-age coordinates of radiometrically dated samples from a stratigraphic section must lie on both the staircase plot of preserved deposits (Fig. 1b) and the original accumulation history (Fig. 1a). Of course, radiometric ages include an unsystematic error and may be plotted as bars, rather than points, to represent a two-standard-deviation or 95% confidence interval about the mean age; but the staircase plot and accumulation history curves must still pass through these bars. In contrast, the elevation-age coordinates of dated changes and dated hiatuses lie on the staircase plot but not necessarily on the accumulation history. The relationship is reversed when active deposition is monitored: a sedimentologist may record the level of the sediment-fluid interface at any time, whether or not sediment at that surface is ultimately preserved. During periods of net erosion, these dated levels record the coordinates of points that lie on the accumulation history but not on the staircase plot.



**Figure 9.** Comparison of mean accumulation rates from levels dated during deposition (trains of circles a-a' and d-d'; i.e. real time sedimentologic observation) and horizons dated in the final deposit (zones b-b' and c-c'; i.e. subsequent stratigraphic sampling). Curves a-a' and b-b' sample a sine wave model of periodic accumulation. Curves c-c' and d-d' were compiled from a Brownian motion model of episodic accumulation. Values of critical parameters for these two models are shown on figures 7 and 8.

What are the consequences of mixing these different dating techniques in one plot of rate against time span? To answer this, the numerical models presented above were sampled according to four different protocols: 1) dated horizons may not lie in a hiatus (e.g. the dated rocks of radiometric techniques); 2) dated horizons must lie in a hiatus (e.g. sequence stratigraphy); 3) dated horizons may lie in a hiatus or a sediment increment (e.g. biostratigraphy, magnetostratigraphy, and isotope stratigraphy); and 4) dated "horizons" must record the level of the sediment surface before any subsequent erosion (e.g. surveyed levels). The first and second protocols are precluded from dating the treads and risers of the staircase plot respectively. The third protocol can sample anywhere on the staircase. The fourth can sample anywhere on the accumulation history; it can represent the staircase risers, but not the treads.

Obviously, the four sampling protocols cannot be expected to produce significantly different rates except for time scales at which hiatuses are prevalent. The Brownian Walk model of random, episodic sedimentation generates hiatuses at all time scales (Figs. 8, 9). All the dating protocols reproduce the -0.5 slope in the regression of rate upon time span that was predicted by theory for a thoroughly random process [22]. The rates calculated from radiometrically dated deposits are systematically 1.5 to 2.0 times faster than those determined from protocols that may sample within a hiatus.

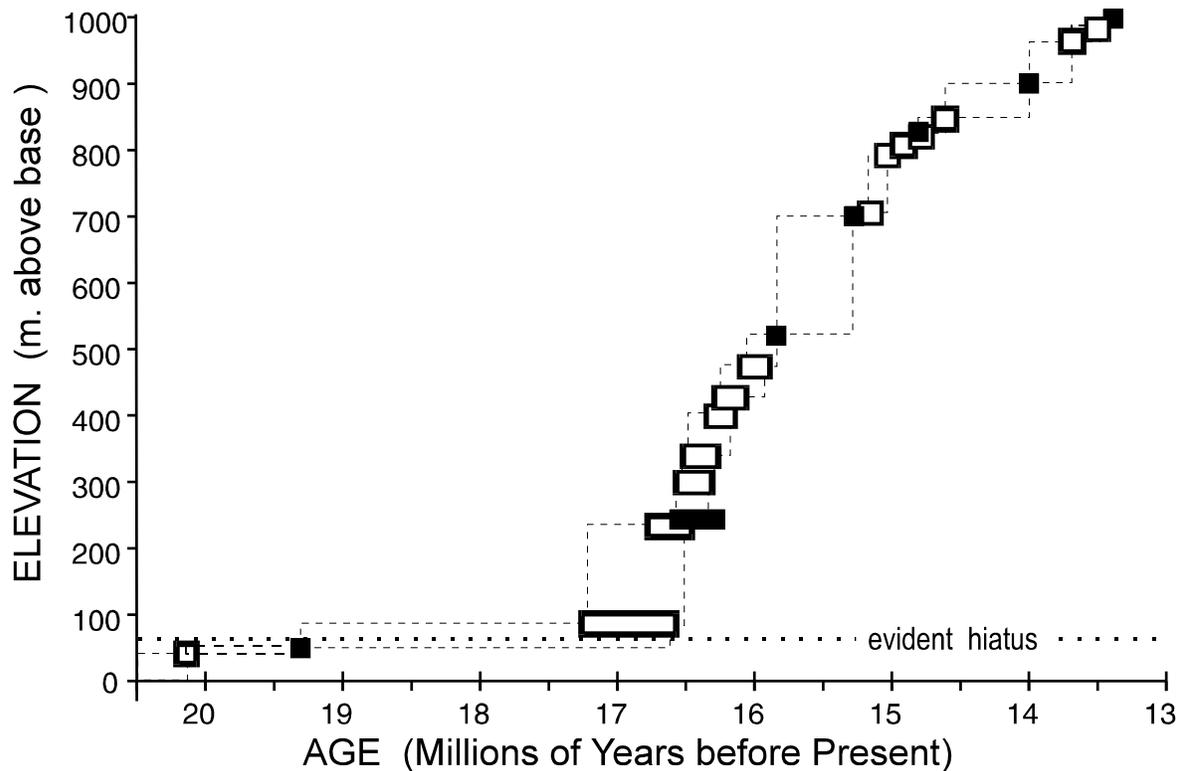
The sine-wave model of periodic sedimentation produces regularly spaced surfaces of hiatus that all share the same time scale. Dated changes and dated hiatuses produce approximately the same estimates of rate for this model (Figs. 7, 9). In Figure 7, radiometrically dated deposits appear to produce a different relationship for rate and time span because this technique has failed to generate any rates at the time scale of the hiatuses; curve a-a' in Figure 7 reproduces the same near horizontal outer segments of the rate-to-timespan relationship that were recovered by dating changes and hiatuses (Fig. 7b and 7c); but curve a-a' lacks points on the steep middle segment. This gap results because the simple sine wave model generates a train of absolutely identical hiatuses. Less simplistic models would include additional periodic components or a stochastic component which would eliminate such strict regularity and allow even radiometric dating of deserts to produce a few rate estimates that lie on the steep middle segment.

Where accumulation rates are determined by direct observation during deposition, however, the models suggest that the relationship of rate to time span (Fig. 9) may depart significantly from stratigraphically determined rates. This departure will be most marked where accumulation is characterized by regular cycles in which a large fraction of the depositional phase is eroded -- a situation which results if the amplitude of the cycles is more than twice the net subsidence achieved in a single cycle period. The sine-wave model in Figures 7 and 9 created such a condition because the wave amplitude was set very large relative to the trend component. The rates determined by direct observation of the depositional process differ from those measurable in the stratigraphic section in two respects. First, direct observation records higher rates because the steepest part of the sine wave is in the mid-section of the accumulation phase; this portion is lost if erosion removes more than half of the deposit of each cycle. Second, direct observation records high rates into longer time spans. The depositional phase lasts for half the wave period; but the time span of the sediment increment that survives erosion can be much shorter.

Even if all rates are determined by dating horizons in a deposit rather than monitoring the active depositional surface, it is still possible that the deposit is too young to be representative of the eventual stratigraphic record [23]. Rates measured before the deposit has aged beyond the return period of the dominant erosional events, do not capture the impact of those events; so they may not be representative of similar time spans in older deposits. If a deposit is older than the return time of the erosional events that determine the stratigraphic architecture, then the deposit may be considered to have "matured" [23], in the sense that it may now be representative of older units of the same time span.

The previous paragraph cautions that the age and the time span of the measured deposit need to be considered when selecting accumulation rates to model the stratigraphic record. Consider the example of marine Holocene deposits on passive continental margins. The Holocene typically

presents the deposits of a single incomplete marine accommodation cycle, one that has yet to pass from a phase of rising sea-level to a phase of falling sea level and become susceptible to exposure and possible erosional losses. Thus, monitoring Holocene accumulation may lead to estimates of accumulation rates on the scale of ten thousand years that are too high to be representative of the corresponding preserved stratigraphic record. The Holocene values may be always be used in models that require the *potential* accumulation rates. Whether or not they represent the net rates of accumulation after low stands of sea level have exposed sediment surface depends upon the ease with which the sediment may be eroded during falling sea level. Obviously, erosion very rarely allows the sediment surface to track the falling sea level closely; usually, the coastline shifts and sedimentary facies change because erosion and accumulation lag behind the destruction and creation of accommodation space.



**Figure 10.** Constraints on the staircase plot for the Barstow Formation, Miocene, southern California. Solid black squares and rectangles are radiometrically dated rocks; heavy open squares and rectangles are paleomagnetically dated levels. The width of the rectangles represents ambiguity in the magnetostratigraphy or multiple radiometric age determinations. Stippled rectangles are the limits of possible paths of the staircase plot.

### A Single Stratigraphic Section

Studies of the type section of the Barstovian Land Mammal Age of North America [24,25] have applied both paleomagnetic and radiometric techniques to date horizons in a single depositional succession (Table 4). Because the stratigraphic section is built of lacustrine and alluvial sediments, which include mudstones, sandstones and coarse conglomerates, it is evident that the flow power of some depositional events would have been adequate to erode previously deposited sediments. Some of the hiatuses marked by bedding surfaces in the section may reasonably be expected to include intervals of erosion. Thus, the section worth examining for differences in the measured accumulation rates that may reflect the dating techniques. And it is a good example to illustrate the difference between the accumulation history and the stratigraphic staircase plot.

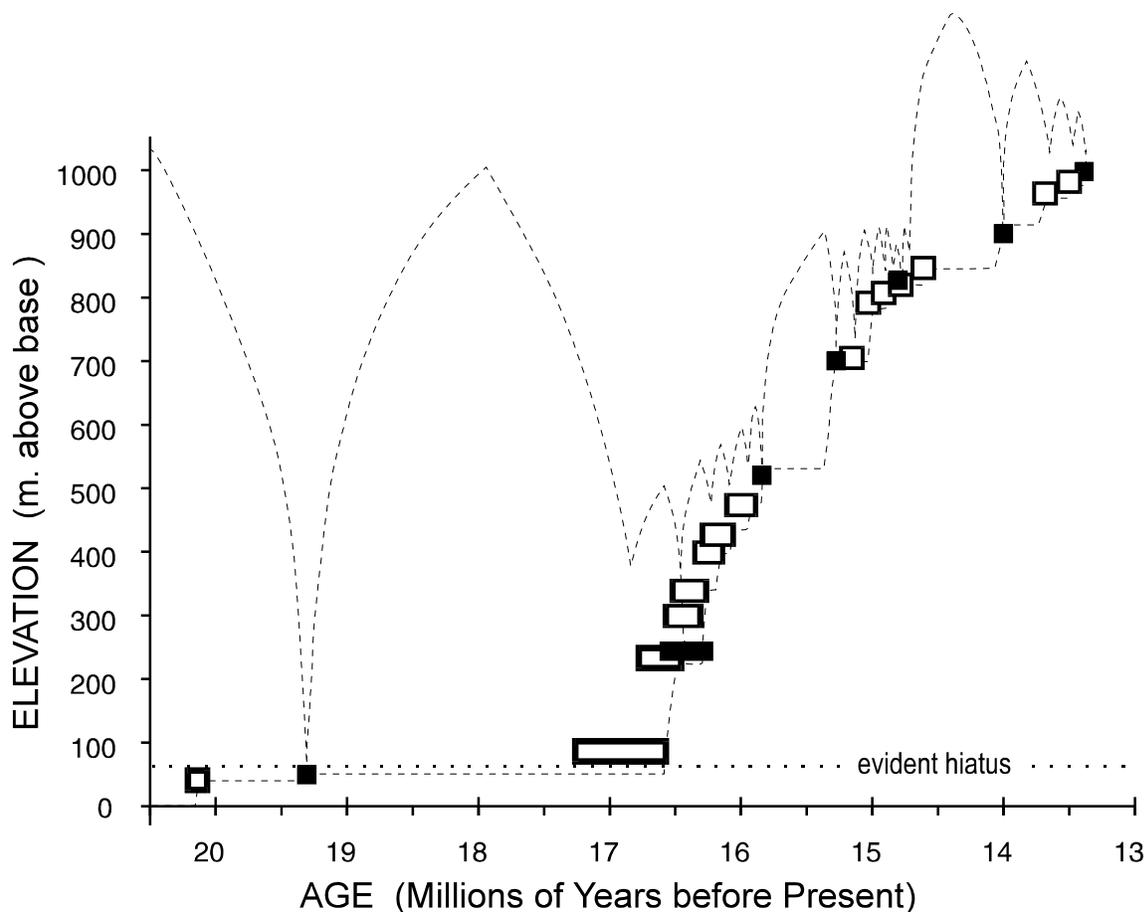
**Table 4** Dated levels in the Barstow Formation

LEVEL (METERS ABOVE BASE )	RADIOMETRIC AGE (MYR)	PALEOMAG. AGE (MYR)	ALTERNATIVE PALEOMAG.	DATED EVENT
1000	13.4			Lapilli tuff
985		13.51		C5ABr
965		13.7		C5ABr
902	14			Hemicyon tuff
845		14.61		C5ADr
828	14.8			Dated tuff
822		14.8		C5ADr
808		14.89		C5Bn.1r
790		15.03		C5Bn.1r
708		15.16		C5Br
701	15.27			Valley View tuff
520	15.84			Oreodont tuff
472		16.01		C5Br
430		16.29	16.1	?C5Cn.2r
401		16.33	16.2	?C5Cn.2r
342		16.49	16.29	?C5Cn.2r
300		16.56	16.33	?C5Cn.2r
245	16.3			Rak tuff
245	16.56			Rak tuff
233		16.73	16.49	?C5Cr
70		17.28	16.56	?C5Cr
50	19.3			Red tuff
38		20.13		C6r

All the dated horizons in the type Barstovian section serve to constrain the location of the staircase plot (Fig. 10). The degrees of freedom to interpret the true staircase plot are given in figure 10 by a chain of small dashed rectangles which meet at the dated horizons. If there is no ambiguity in the age of a horizon, the lower left corner of each rectangle touches the upper right corner of the next. Otherwise, the upper and lower edges overlap.

The accumulation history curve is significantly less well constrained than the staircase plot for any ancient stratigraphic section. As explained when the different graphics were introduced, the staircase plot and the coordinates of dated horizons tend to identify the lower bound on the accumulation history curve (Fig. 11). Of course, the curve must pass through the elevation/age coordinates of the radiometrically dated horizons. But the paleomagnetically dated changes might lie at a hiatus. In other words, they might lie on a horizontal tread of the staircase plot and the tread may extend to the left and right of the paleomagnetic age. If so, the accumulation history curve need not pass through these “dated” coordinates; it is free to ascend to the left of the dated point, pass through a maximum, and then descend to the right of the dated point; the descent must end in a minimum at the level of the dated change. Such maxima, skirting over the dated changes represent temporary deposition followed by erosion (see Fig. 3b); they are the asymmetrical portions of the accumulation history that depict hiatuses and are missing from the staircase plots.

Figure 11 attempts to place upper limits on the size of the missing maxima and, thus, place an upper bound on the accumulation history. The missing maxima appear as a set of dashed gothic arches



**Figure 11.** Constraints on the accumulation history of the Barstow Formation. Black rectangles: radiometrically dated rocks; open rectangles: paleomagnetically dated levels; Dashed arches: limits of possible paths of the accumulation history curve, which must pass through the dated tuffs, but may include a hiatus in the intervals of paleomagnetic reversal.

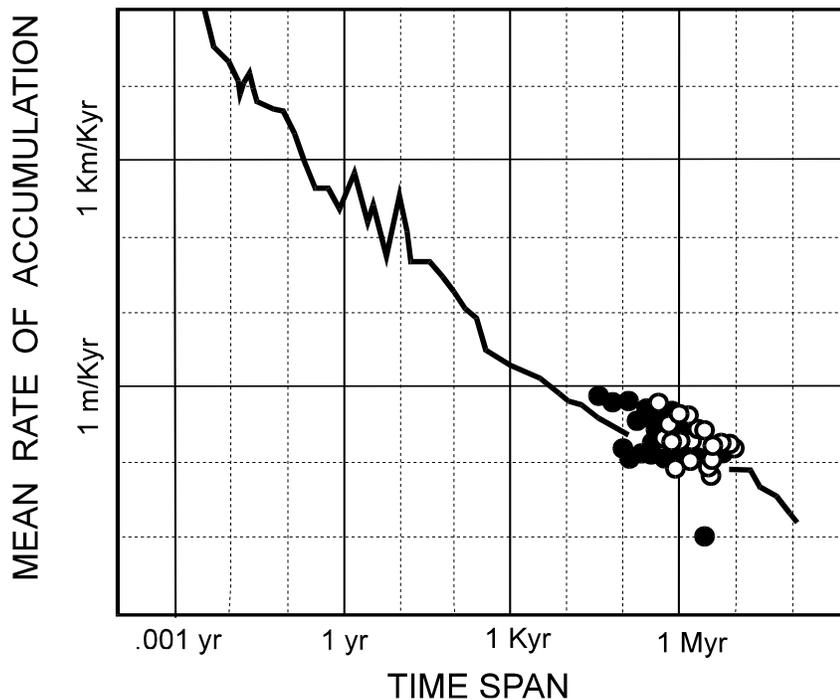
over the dated coordinates of paleomagnetic events. The critical task is to determine the *highest credible* arch. Two types of information constrain the answer: the width of the arch must be consistent with the neighboring dated coordinates up and down section; and the steepness of the arch must represent credible accumulation rates. Let us consider accumulation rates first.

We have seen that accumulation rate can be immeasurably high in the very short term; so the beginning of each arch can be almost vertical. But very high rates are not sustained for long intervals; rate scales inversely with time span and the steepness of the arch must decrease upward. Remember that the arch is not the accumulation history curve, it is its upper bound. Although short intervals of rapid accumulation can occur at any time, the arch is attempting to draw the upper limit of all reasonable curves. Thus, its form is guided by the maximum sustainable rates as a function of time span since the beginning of the arch.

As a guide to the rate of decrease in the steepness of the side of the arch, I used the accumulation rates determined in the same section. Because the scales in figure 11 are not logarithmic, the power laws lead to the progressively flattening curve. For lack of any specific information on erosion rates, I made the descending sides of the arches symmetrical with the ascending sides. Using local field relationships, Friend and others have drawn accumulation histories [26: Fig. 1e, 3e] in which the erosional side of the arches are very steep and the tops flat. Again, the arches here are not a

guess at the corresponding segment of the real accumulation history but an attempt to place an upper envelope around all reasonable guesses. The flat-topped curves drawn by Friend and others fit inside the arches drawn in figure 11.

The width that each arch can attain is determined by the age and elevation of neighboring dated horizons in the section. Closely spaced dated horizons force the arches to be relatively narrow and this, in turn, limits the height that the arches can reach. The accumulation history must either pass through a magnetostratigraphic coordinate, or execute a minimum before the next dated horizon. Thus, even though the elevation-time coordinates of dated changes need not lie on the accumulation history, the upper limit of the curve is better constrained where a closely spaced succession of changes has been dated.



**Figure 12.** Rates of accumulation in the Barstow Formation determined by radiometric dating (open circles) and magnetostratigraphy (filled circles). These are compared with the trend of mean rates for fluvial and alluvial deposits (solid curve) based upon 12,338 empirical determinations.

The regression of rate upon time span is difficult to determine reliably for the type Barstovian section alone because available data span only two and a half orders of magnitude in time span. Visual inspection of Figure 12 reveals that the mean slope of a regression through the available data is likely to be negative and less steep than  $-0.5$  overall. The slope of maximum rates steepens appreciably at time scales longer than 2 million years. We should guess that accumulation was characterized by positive feedback in depositional phases that lasted up to 2 million years. At longer time scales hiatuses are likely more prevalent than a random walk would predict and the section is significantly less complete than at shorter time scales. There is considerable overlap between the rates determined by radiometric and paleomagnetic techniques, but the maximum rate, at a given time scale, is typically determined by radiometry. If not the result of pure chance, this observation implies that a significant number of the paleomagnetic reversals take place across hiatuses in the type section.

## Conclusions

Sediment accumulation must be acknowledged to be potentially unsteady and discontinuous at all sedimentologically and stratigraphically significant time scales. For this reason, it is safest to regard sediment accumulation as a fractal phenomenon and expect that accumulation rates will be a strong negative function of time scale. Large empirical compilations reveal that negative power laws relate rate to time span as would be expected for a fractal time series.

The steepness of the negative power laws means that much more of the variance in empirical accumulation rates can be explained by the length of the time interval of measurement than any other single factor, including environment of deposition or geologic age. The slope of the power law does vary with depositional environment and time span. The latter observation means that hiatuses are more prevalent at some time scales than others; steeper portions of the negative power laws indicate that a relatively higher fraction of time is apportioned to hiatuses at these time scales than at time scales for which the negative slope is less steep. In shallow marine deposits, a steepening of the negative power law indicates that hiatuses are more prevalent at time scales of tens to hundreds of thousands of years than at shorter or longer time scales. This may reflect the influence of Milankovitch-scale fluctuations in sea level and the prevalence of meter-scale accommodation cycles.

A purely stochastic accumulation process would lead to a negative power law with a slope of -0.5. The discovery of slopes that are both steeper and gentler than -0.5 means that both positive and negative feedback characterizes the process of sediment accumulation. In general, positive feedback is more likely at longer time spans. At time spans longer than a few hundred thousand years the differences in accumulation rate that might be attributed to depositional environment are relatively small. These observations are consistent with the surmise that lithospheric subsidence becomes the critical determinant of sediment accommodation at time scales longer than a hundred thousand years. At time scales shorter than a few thousand years, sediment supply and depositional process more likely determine accumulation rate. The dramatic fall in shallow-marine sedimentation rates at time scales of tens of thousands to hundreds of thousands of years has already been attributed to the dominance of the eustatic component of sediment accommodation at these intermediate time scales.

Where sediment accumulation rates are calculated from pairs of datum levels, hiatuses influence the results in two ways:

- 1) Hiatuses that lie between the two datum levels cause the average accumulation rate to underestimate the sedimentation rate in the intervals between hiatuses. As datum levels become more widely spaced the calculated rates are likely to include more and larger hiatuses. This explains the negative regression of measured rate upon time span, from which some information may be extracted concerning the size and spacing of hiatuses. Because the negative slopes of the regression persist to time spans of at least a hundred million years, we must assume that hiatuses are likely at all time scales up to tens of millions of years.
- 2) A datum level that lies at a surface of erosional hiatus may yield a false elevation-time coordinate in the sense that it does not lie on the true accumulation history. This is a problem for dated changes (e.g. paleomagnetic reversals, biostratigraphic zone boundaries) and dated hiatuses (e.g. sequence boundaries), but not for dated deposits (e.g. ash beds). Comparison of rates determined by different techniques, therefore, offers further insight on the distribution of hiatuses. On the other hand, caution must be exercised when combining rates determined from the stratigraphic record with rates determined by monitoring an active depositional system. The monitoring process can determine rates that characterize phases of temporary deposition; i.e. deposits which are subsequently eroded and are, therefore, not necessarily representative of the longer term stratigraphic record.

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