Simulations of Earthquake Nucleation, Its Static Perturbation, and Aftershock Rates on Faults with Rate and State Friction

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INTRODUCTION

Large earthquakes are usually followed by increased seismic activity that decays to the background rate over time, a process described empirically by Omori's law. Dieterich (*JGR*, 1994) proposed that Omori's law could result from perturbing a population of nucleation sites governed by the laboratory-derived rate and state friction laws and a spring-slider model. The model was further explored in a number of studies (i.e., Gomberg et al., *JGR*, 2000) and used to interpret observations (i.e., Toda et al., *JGR*, 1998). Here we explore the consequences of Dieterich's approach in a 2-D continuum, where the nucleation process can be more complicated than assumed in Dieterich's model. Our approach is different from previous studies of aftershock rates with rate-state friction in that here, nucleation processes are simulated as a part of spontaneously occurring earthquake sequences and hence the initial conditions for the nucleation process are determined by the model itself. We find that the nucleation in a continuum elastic setting proceeds differently in different models.

DIETERICH'S MODEL OF AFTERSHOCK STUDIES (*JGR*, 1994)

Rate and state friction $\quad \tau = \sigma\left[f_0 + a\ln(V/V_0) + b\ln(V_0\theta/L)\right], \quad d\theta/dt = 1 - V\theta/L$

Combined with a single degree of freedom spring slider model $\quad \tau(t) - k\delta = \sigma\left[f_0 + a\ln(V/V_0) + b\ln(V_0\theta/L)\right]$

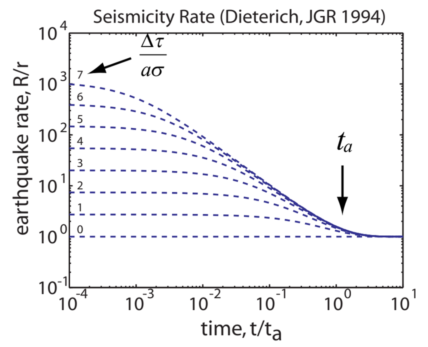
The state variable assumption, $V\theta/L \gg 1$

(θ is behind its steady-state value due to fast acceleration of slip) $\quad \left(\frac{\partial\theta}{\partial\delta}\right)_{\sigma=const} = -\frac{\theta}{L}$ and thus, $\theta = \theta_0 e^{-\delta/L}$, or $\ln(\theta) = Const - \delta/L$

A population of nucleation sites that result in constant background seismicity rate r.

From the Dieterich's formulation, a sudden stress step $\Delta\tau$ would result in different (higher) earthquake rate R (aftershock rate), which can be computed as:

$$\frac{R}{r} = \frac{1}{\left[\exp\left(\frac{-\Delta\tau}{a\sigma}\right) - 1\right]\exp[-t/t_a] + 1}$$



t_a is the aftershock duration, $t_a = a\sigma/\dot{\tau}$ and $\dot{\tau}$ is the background stressing rate.

The resulting aftershock rate follows Omori's law for aftershock decay, $R = \frac{K}{c+t}$, for a range of t.

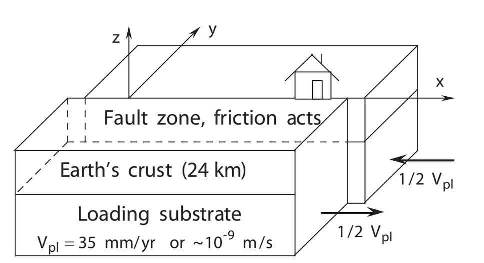
For this model to be consistent with observations, $\Delta\tau/a\sigma$ has to be reasonably large.

Observations, with estimated $\Delta\tau$, suggest $a\sigma$ = 0.01 - 0.1 MPa (i.e., Toda et al., *JGR*, 1998). Assuming normal stresses σ ~ 100 MPa (overburden minus hydrostatic pore pressure at 6-8 km depth), this translates into a ~ 0.0001-0.001, smaller than typical laboratory values a ~ 0.01. That is why we consider different values of a.

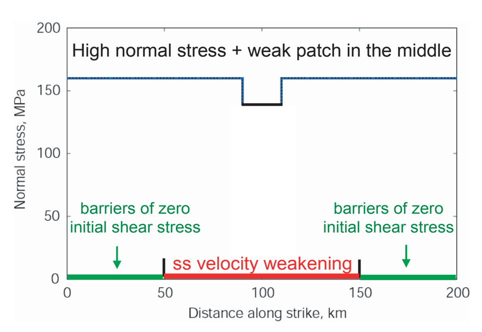
FAULT MODELS

The 3D model geometry exhibits a planar strike-slip fault embedded into the elastic half space and loaded by the substrate below with the relative motion of 35 mm/year. In our study, we use two simplified 2D models. The model and the simulation methodology are described in Lapusta et al. (*JGR*, 2000), Lapusta and Rice (*JGR*, 2003).

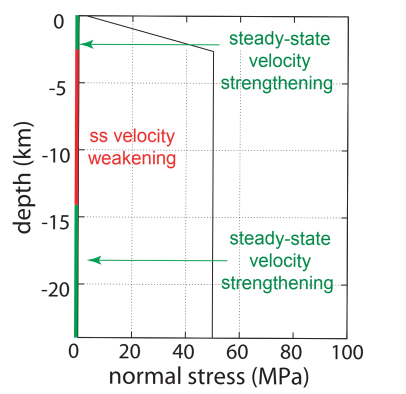
3D Geometry



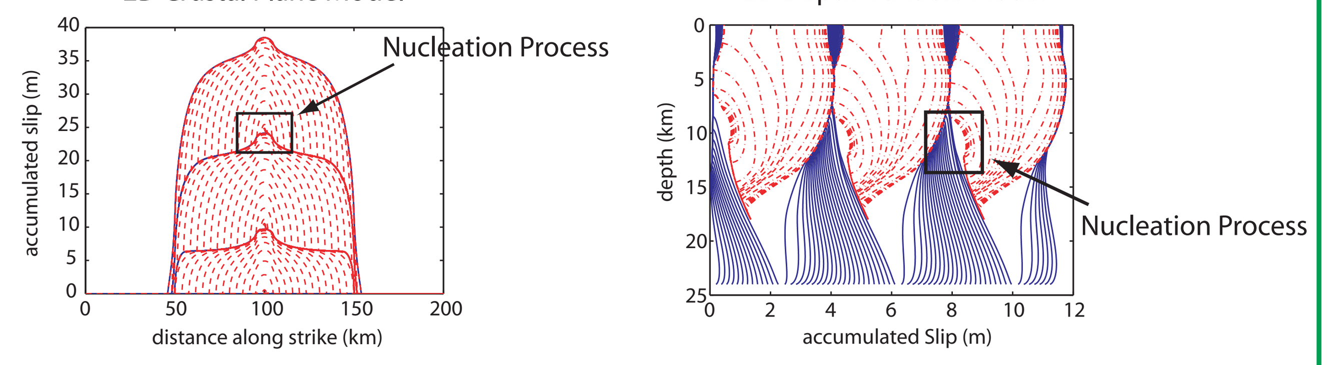
2D Crustal Plane Model



2D Depth-Variable Model

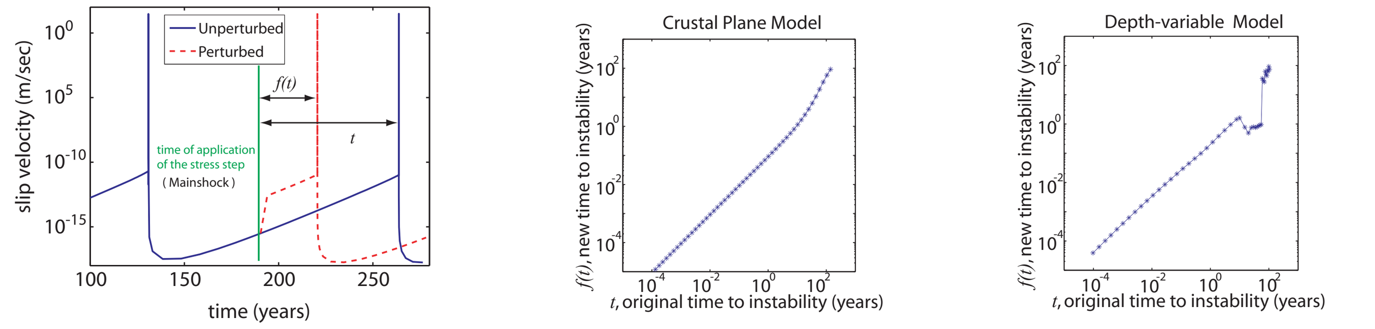


SIMULATIONS OF EARTHQUAKE SEQUENCES



RESPONSE TO STATIC STRESS CHANGES

We perturb our simulations during nucleation with a static stress increase and simulate the subsequent progression of the nucleation and the following dynamic event. This procedure is repeated at different stages of the nucleation process and allows us to obtain the function *f(t)*, where *t* is the original time to instability for the nucleation process and *f(t)* is its new time to instability.



AFTERSHOCK RATE CALCULATIONS

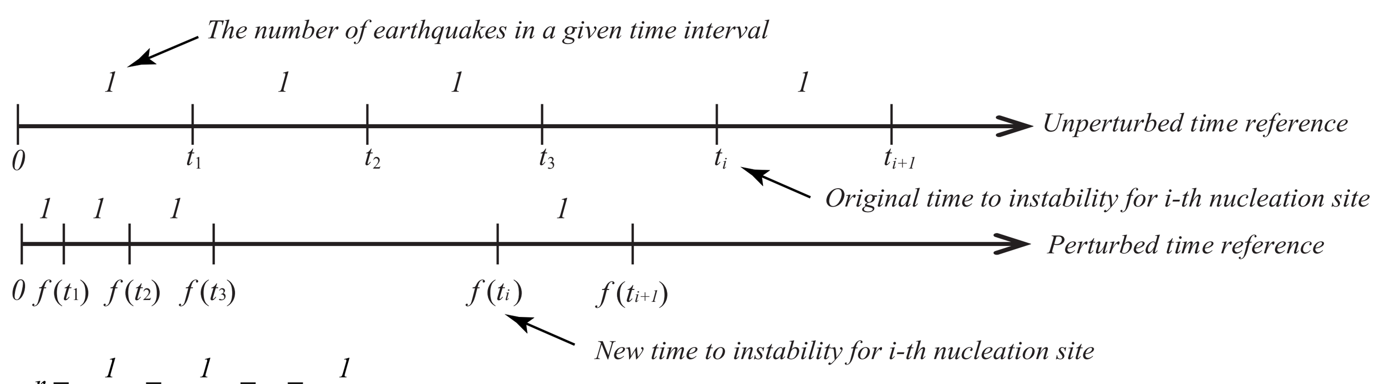
We use the numerically obtained function *f(t)* to calculate the corresponding aftershock rate.

Important assumptions, following Dieterich's approach:

(1) A population of nucleation sites, each of them following the behavior observed in our simulations.

(2) At the time of the static stress change (or mainshock), each of the nucleation sites is at a different stage in the nucleation process, so that, if left unperturbed, the population of the nucleation sites would produce earthquakes at a uniform rate.

After the static stress change, the time to instability of each nucleation site changes, resulting in a different earthquake rate (aftershock rate) which we compute as follows:



$$r = \frac{1}{t_2 - t_1} = \frac{1}{t_3 - t_2} = \dots = \frac{1}{t_{i+1} - t_i}$$

$$R[f(t_i), f(t_{i+1})] = \frac{1}{f(t_{i+1}) - f(t_i)} = \frac{r(t_{i+1} - t_i)}{f(t_{i+1}) - f(t_i)}$$

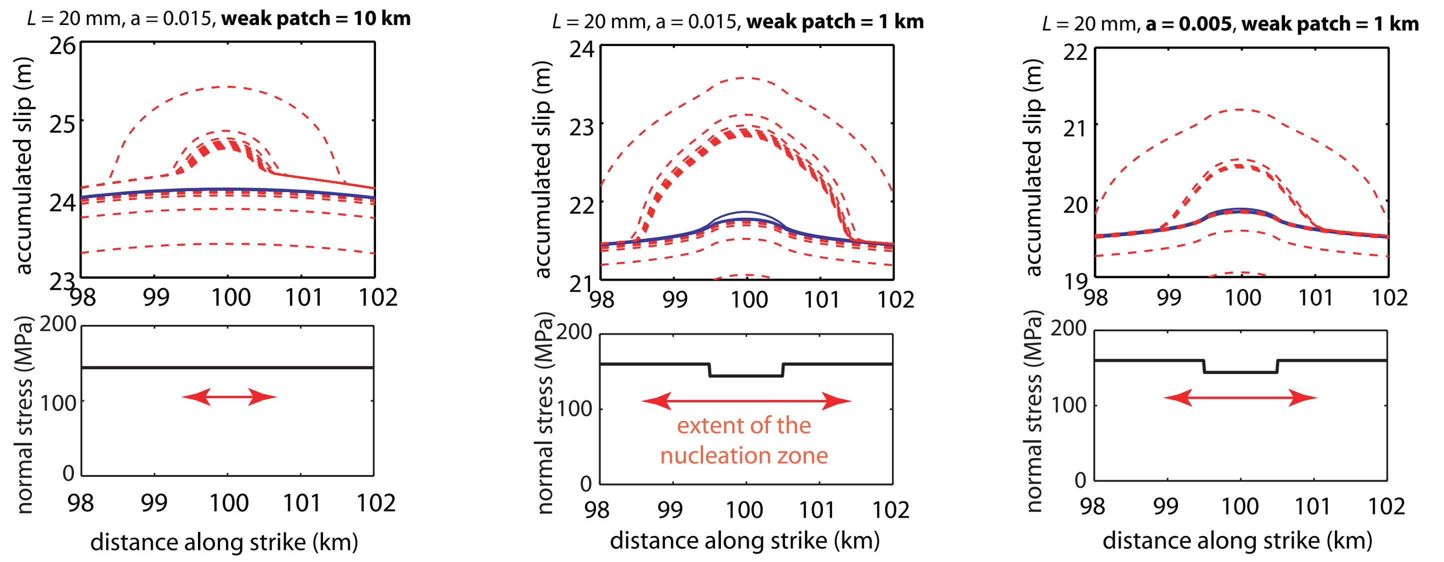
$$\frac{R}{r}[f(t_i), f(t_{i+1})] = \frac{t_{i+1} - t_i}{f(t_{i+1}) - f(t_i)}$$

Differences with the Dieterich's approach:

(1) There are no simplifying assumptions about rate and state friction behavior.
(2) We use elastic continuum settings and not spring-slider block model.
(3) We study nucleation processes simulated as a part of spontaneously occurring earthquake sequences and hence the initial conditions for the nucleation process are determined by the model.
(4) Our results are obtained numerically and not analytically as was possible for the Dieterich's model.

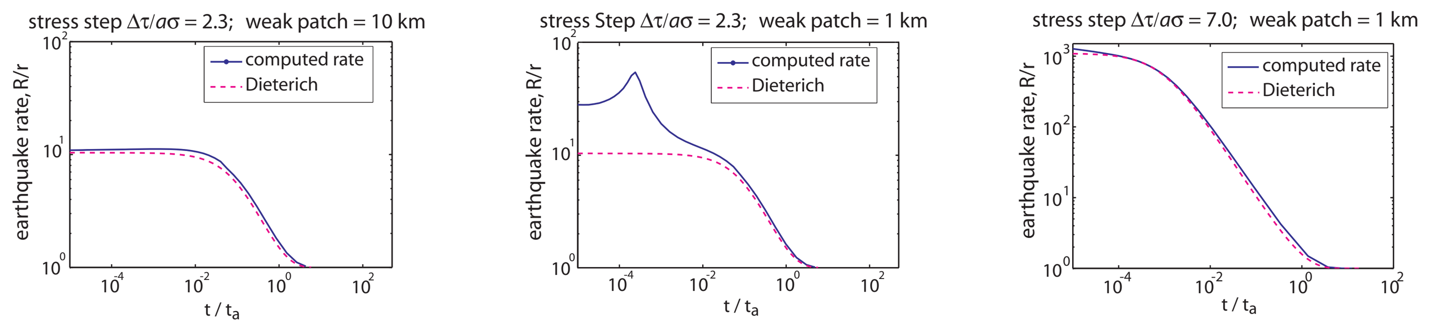
NUCLEATION PROCESSES IN THE CRUSTAL PLANE MODEL

CASE 1	CASE 2	CASE 3
Relatively homogeneous stress conditions	Heterogeneous normal stress within the nucleation region	Same as CASE 2 except smaller value of *a*



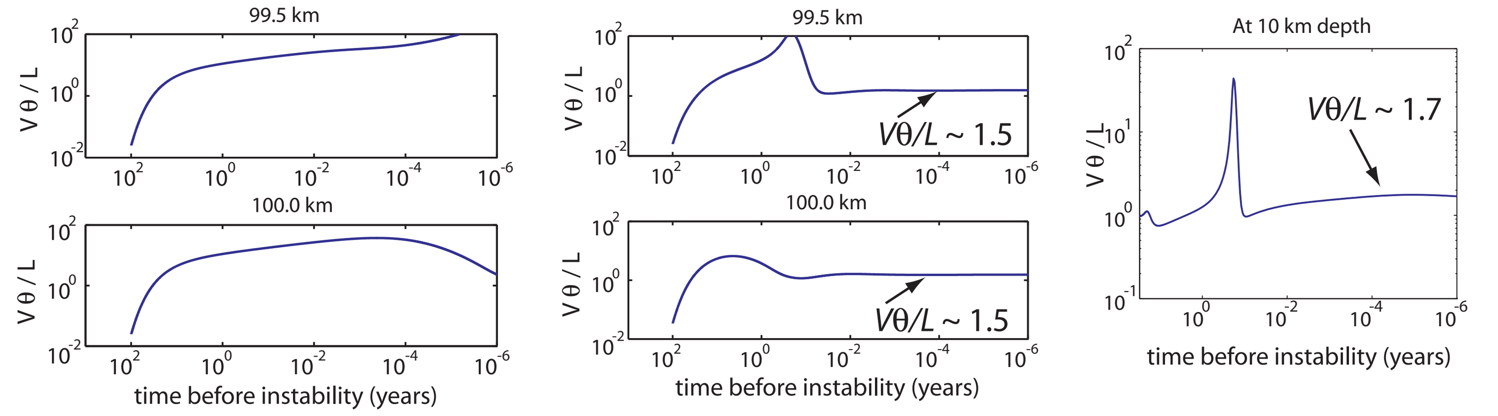
The dashed red lines are plotted every second if the maximum slip velocity on the fault exceeds 1 mm/sec, whereas the blue lines are plotted every 20 years. **The nucleation processes and the final sizes of the nucleation zone vary with the size of the weak patch and the parameter *a***, with the same (*b* - *a*).

AFTERSHOCK RATES FOR THESE NUCLEATION PROCESSES



Aftershock rates based on our simulations follow the Dieterich's analytical solution in most cases. However, some cases, such as CASE 2 (middle figure), have extra features. Note that CASE 2 also has heterogeneous normal stress within the nucleation region. Increasing $\Delta\tau/a\sigma$ (by decreasing *a*) results in higher earthquake rates as in Dieterich. It also eliminates the extra peak (CASE 3, right figure). In the plots, the time is normalized by t_a which is the aftershock duration in the Dieterich's model.

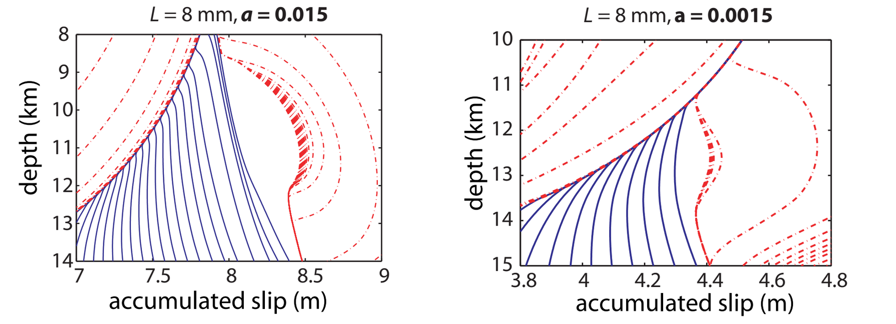
VERIFYING THE ASSUMPTION $V\theta/L \gg 1$



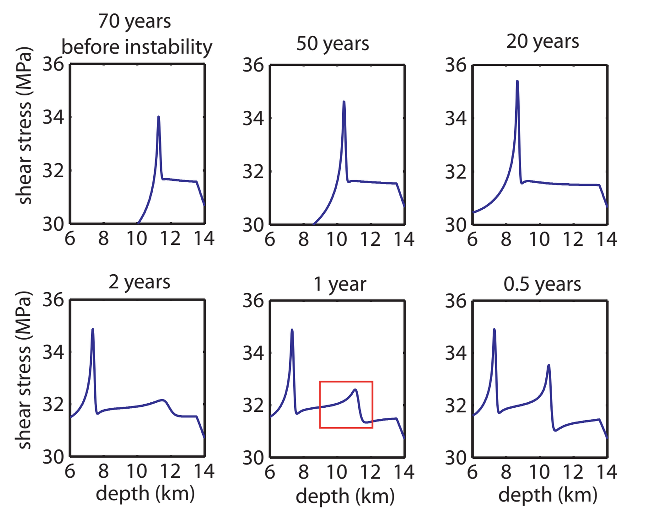
Quantity $V\theta/L$ as a function of time before the dynamic event at different points inside the nucleation region. **Dieterich's state variable assumption is valid in CASE 1 of relatively homogeneous stress** conditions, as $V\theta/L \gg 1$ throughout the nucleation process. The assumption is **invalid in CASE 2 of heterogeneous normal stress**, starting at ~0.1 years before the dynamic event and resulting in a different nucleation process and thus the peak in the aftershock rates. $V\theta/L \gg 1$ is **invalid for much of the nucleation process in a transition zone** (CASE 4) where the shear stress is heterogeneous in the nucleation zone due to the creeping region.

NUCLEATION PROCESSES IN THE DEPTH-VARIABLE MODEL

CASE 4	CASE 5
Transition between creeping and locked behavior	Same as CASE 4 except smaller value of *a*

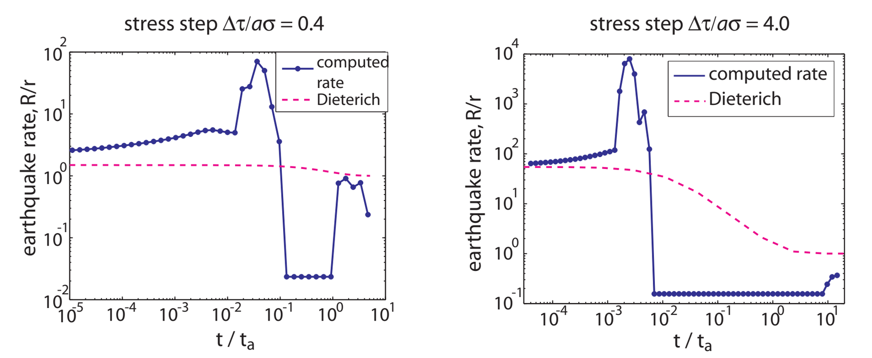


Shear stress at the nucleation region (9 - 12 km) is heterogeneous before dynamic event (CASE 4)



Non-uniform rheological properties with depth result in nucleation at the transition between creeping and locked regions. **The nucleation proceeds in temporally and spatially non-uniform stress field caused by the stress concentrations due to the nearby creeping region.** In the model with smaller *a* (CASE 5), the nucleation process happens quicker and the instability develops more abruptly.

AFTERSHOCK RATES FOR THESE NUCLEATION PROCESSES



The resulting aftershock rates exhibit pronounced delayed peaks and further oscillations. This response is very different from the Dieterich analytical prediction. Smaller values of *a* (and hence larger relative value of the stress step) do not eliminate the peak in this model.

CONCLUSIONS, DISCUSSION, AND FUTURE WORK

Nucleation processes in 2D continuum models and the resulting aftershock rates are well-described by the model of Dieterich, 1994 when the stress conditions in the nucleation region are relatively uniform. In that case, Dieterich's assumption that the state variable is significantly behind its steady-state value holds for most of the nucleation process.

On the contrary, aftershock rates in models with heterogeneities within the nucleation zone exhibit behavior different from Dieterich's model, with delayed peaks of aftershock activity, and the state variable assumption is violated for a significant portion of the nucleation process. The heterogeneities we considered are in the form of a patch of lower normal stress or due to interactions with creeping regions at rheological transitions.

Note that in our aftershock rate calculations, all nucleation sites experience the same stress step, while in a more realistic model the stress step should vary depending on where the site is located in relation to the mainshock.

This study implies that it is important to understand whether the stress conditions are homogeneous on the scale of aftershock nucleation sizes. If they are, then the model produces Omori's law in a certain parameter range. Otherwise, the model does not result in Omori's behavior and needs to be modified. For example, it may be possible to construct the observed Omori's decay of aftershocks by assuming different parameters for different nucleation sites.

Our future work will be directed towards:

(1) **Simulating earthquake nucleation in a 3D model** in order to verify our conclusions about the features of nucleation processes occurring in homogeneous and heterogeneous conditions.

(2) **Constraining models of earthquake nucleation based on observations**, by looking for the evidence of delayed triggering observed in our models, by trying to isolate a subset of aftershocks occurring at rheological transitions, and by considering earthquake triggering by tides.